# Guided waves in pre-stressed hyperelastic plates and tubes

Guo-Yang Li[1], Qiong He[2], **Robert Mangan**[3], Guoqiang Xu[1], Chi Mo[1], Jianwen Luo[2], Michel Destrade[3], Yanping Cao[1]

[1]Institute of Biomechanics and Medical Engineering, Department of Engineering Mechanics, Tsinghua University, Beijing, PR China

[2]Department of Biomedical Engineering, School of Medicine, Tsinghua University, Beijing, PR China

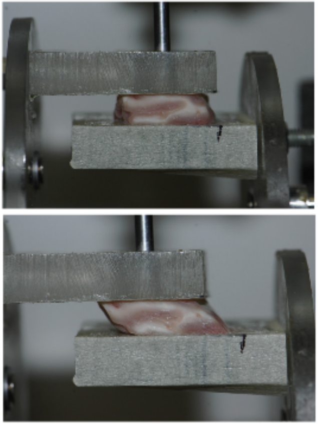
[3]School of Mathematics, Statistics and Applied Mathematics, National University of Ireland Galway, Ireland

NUI Galway
OÉ Gaillimh

清華大學
Tsinghua University

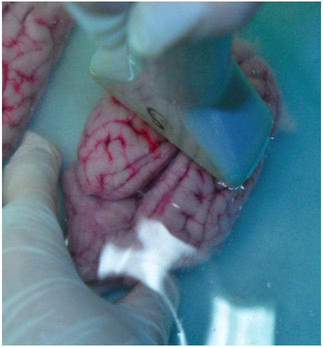
# Mechanical properties of biological tissues

- Useful for diagnostics or for simulations (e.g. stents, head impacts)
- Found from **mechanical tests**, treating tissues as an engineering material.
- **Destructive**, can't be applied *in vivo*



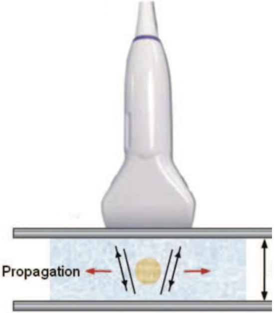
# Mechanical properties of biological tissues

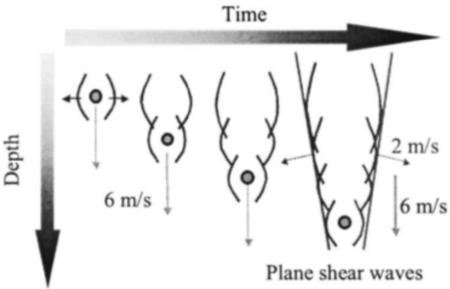
- Alternatively, use **non-destructive acoustic waves**
- Speed of the wave depends on the material properties
  - ➧ Measure wave speed to infer mechanical properties
- Can apply *in vivo*



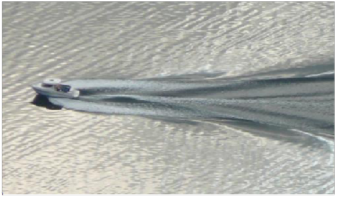
# Generating waves

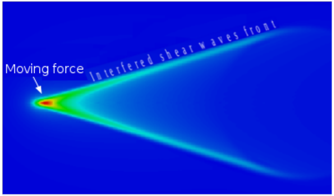
- Probe generates a real-time **ultrasound image**
- Also generates a low frequency **shear wave** by focusing acoustic beam
- The wave is seen in the ultrasound field and its **speed is measured**



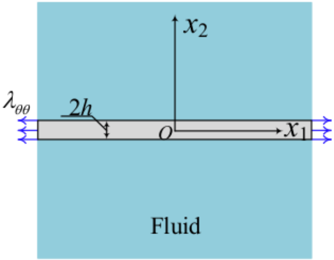
# Elastic Cherenkov effect

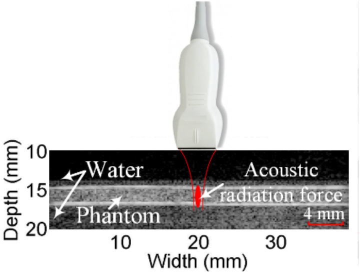




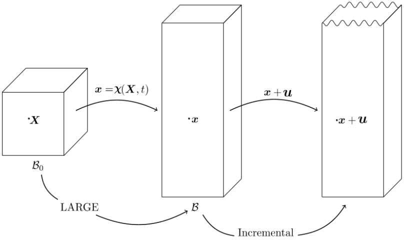

Three similar wave phenomena observed when the velocity of the excitation source is greater than the velocities of the resulting waves in the media.

Now we consider **guided waves** (generated using Verasonics device) in a **stretched** polyvinyl alcohol (PVA) cryogel plate immersed in water.



Model wave propagation in solid as small "incremental" deformation superimposed on a large deformation.




Equations of motion:

$$\nabla \cdot \boldsymbol{\Sigma} = \rho \ddot{\mathbf{u}} \qquad \text{(solid)}$$

where $\boldsymbol{\Sigma} = \boldsymbol{\mathcal{A}}_0 \nabla \mathbf{u}$ and $\boldsymbol{\mathcal{A}}_0 = \frac{\partial^2 W}{\partial \mathbf{F} \partial \mathbf{F}}$,

$$\nabla(\kappa \nabla \cdot \mathbf{u}^F) = \rho^F \ddot{\mathbf{u}}^F, \qquad \text{(fluid)}$$

where $c_p = \sqrt{\kappa/\rho^F}$ is the speed of sound in the fluid.

Seeking a wave solution $e^{skx_2}e^{ik(x_1-ct)}$, and imposing continuity of stress and displacements across the fluid-solid interfaces, we find both symmetric and anti-symmetric solutions.

For the anti-symmetric mode, the **dispersion equation** reads

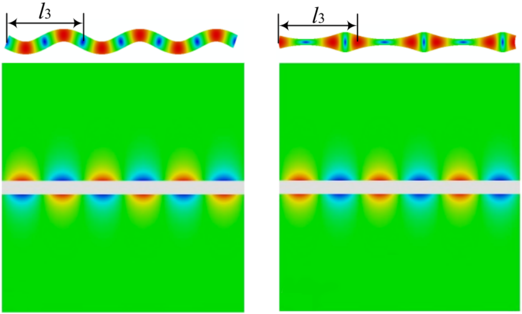
$$\gamma s_1(1+s_2^2)^2\tanh(s_1 kh) - \gamma s_2(1+s_2^2)^2\tanh(s_2 kh) + \frac{\rho^F c^2}{\sqrt{1-\frac{c^2}{c_p^2}}}(s_1^2 - s_2^2) = 0.$$

When the plate is not stretched, we recover the equation [3]

$$\left(2-\frac{\rho c^2}{\mu_0}\right)^2\tanh(kh_0) - 4\sqrt{1-\frac{\rho c^2}{\mu_0}}\tanh\left(\sqrt{1-\frac{\rho c^2}{\mu_0}}kh_0\right) + \frac{\rho\rho_F c^4}{\mu_0^2\sqrt{1-\frac{c^2}{c_p^2}}} = 0,$$

where $\mu_0$ is the shear modulus.

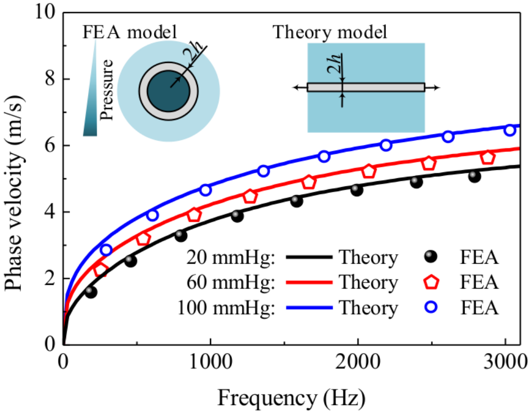
# Finite element simulations




Anti-symmetric and symmetric modes

# Finite element simulations

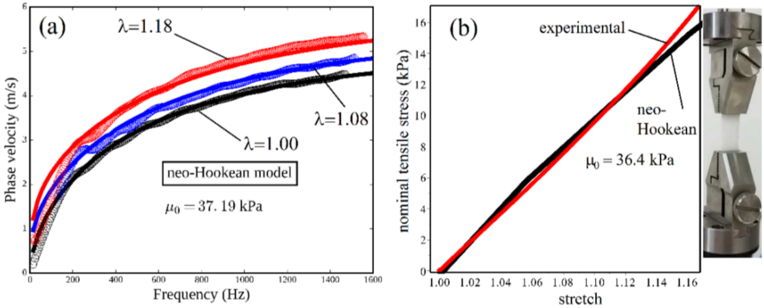
- For large radius-to-thickness ratio, FE simulation of waves in a tube agree with theoretical **plate** model.
  - ➡ Can use plate theory for tubes (e.g. arteries)



# Curve fitting

Determine material parameters by fitting the theoretical curves to the experimental data. For example, the **neo-Hookean model** was used:

$$W = \frac{\mu_0}{2}(I_1 - 3). \quad (1)$$




(a) dispersion curves at various stretches, (b) stress response in destructive tensile test

# References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