

# Flocking models

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We shall investigate flocking models from the perspective of multi-agent consensus.

# Consensus

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# Flocking models

The modelling of flocks has been an active area of applied mathematics for the past twenty years. One of the first breakthroughs occurred in the 1980's, when Craig Reynolds used a simple set of algorithms to generate realistic computer models of flocking agents, which he called *boids*.

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Vicsek et al. developed *self-propelled particle models* which incorporated disturbances/noise to the swarms whereas Kennedy and Eberhart focussed on optimisation models to simulate the complex choreography of flocking animals.

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- track your neighbours
- don't crash

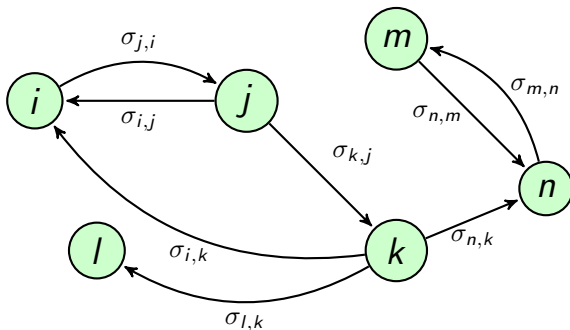
# Networks

A *network* is a weighted graph, that is, a set of elements called *nodes* or *vertices*, which may be connected to one another via relational links (*edges*). To each node we assign a *state* and to each edge a weight (or *gain*),  $\sigma_{ij}$ .



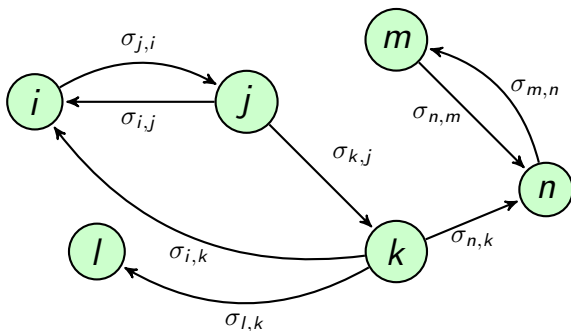
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We want our states and gains to evolve until consensus is achieved.

# Differential equations

The state and gain evolutions are governed by a system of coupled differential equations. The general form being:

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We aim to tailor equations (1) and (2) to effect flocking in a multi agent model. Some features to be incorporated in the gain based model are



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## References

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