Building Sparse Graphs by Inductive Operations

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Outline

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- Some Inductive operations.
- Some well-known characterisations of sparse graphs.

The Main Features of Inductive Construction

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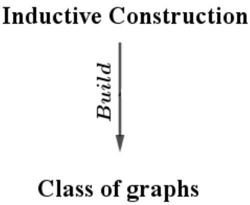

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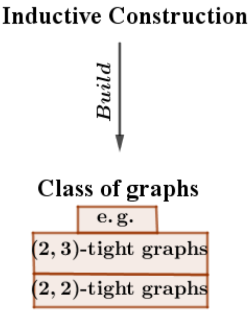

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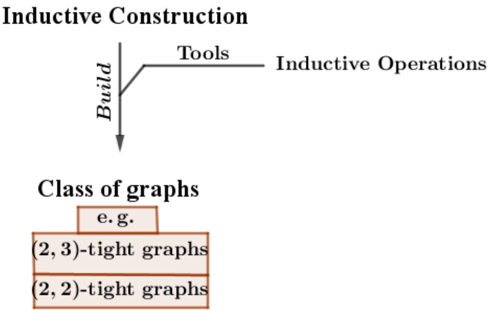

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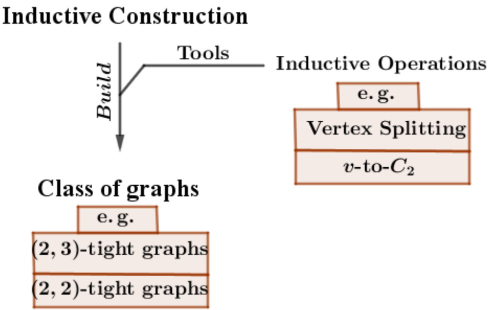

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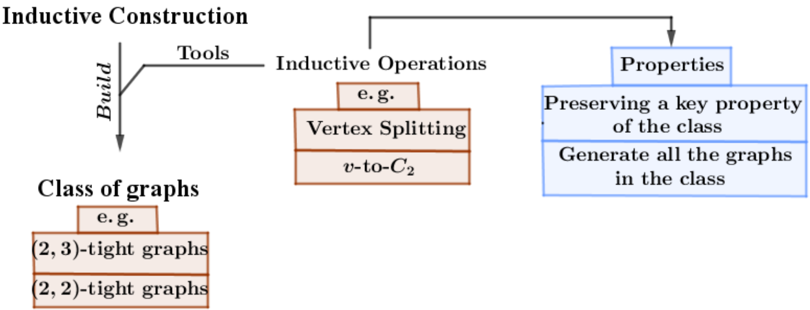

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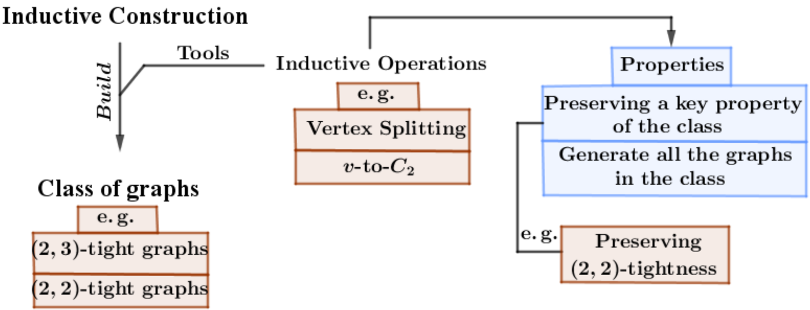

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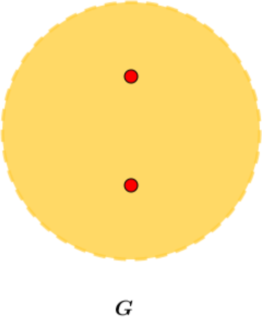
- G is called (k, l)-tight if G is (k, l)-sparse and $|E| = k|V| - l$.

Some Inductive operations

Henneberg Operation H_0



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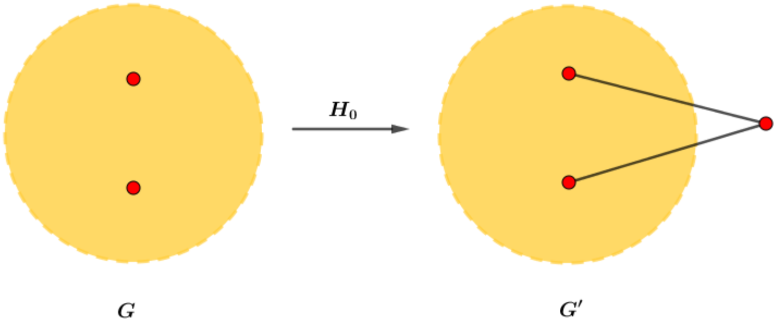
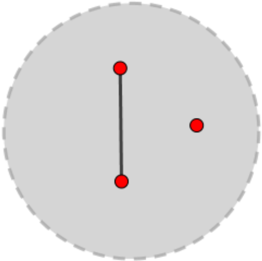


Henneberg Operation H_1



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Some Inductive operations

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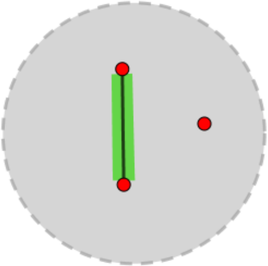

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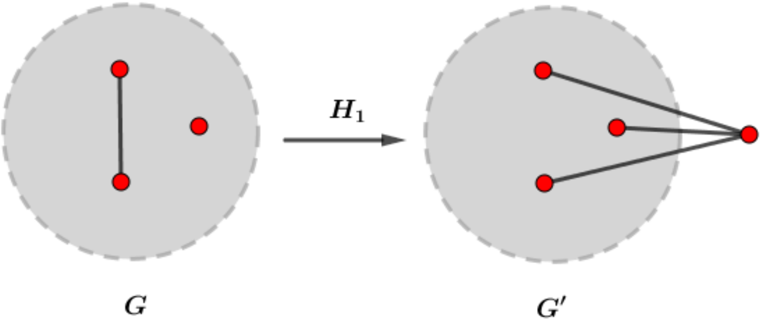
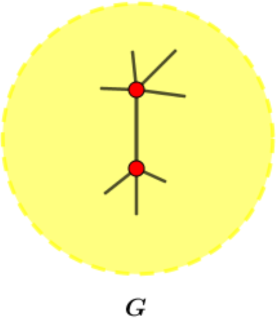


Vertex Splitting Operation



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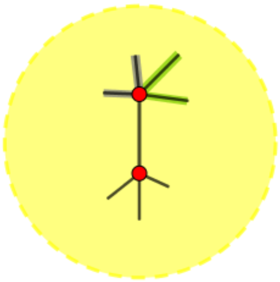

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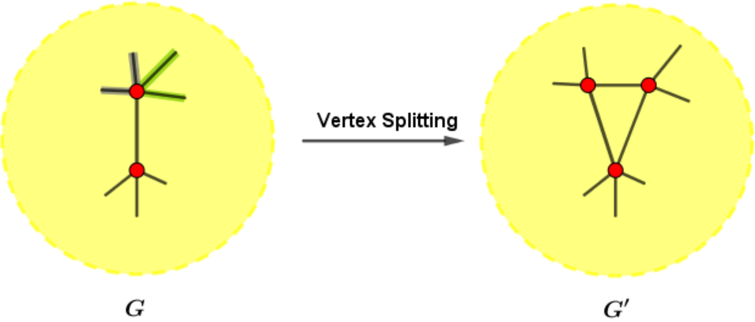
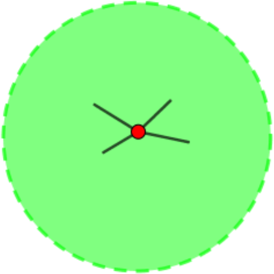


v-to-C_2 Operation



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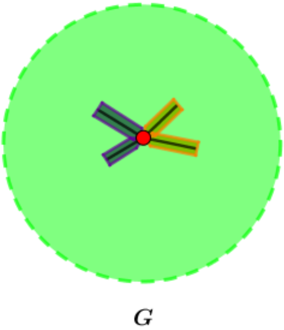
v-to-C_2 Operation



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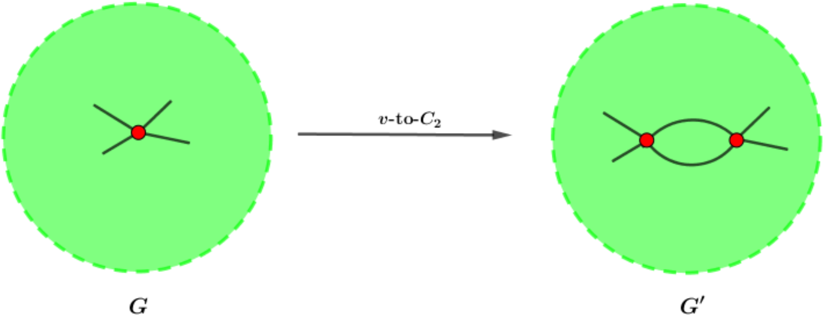
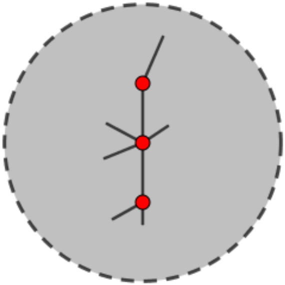


Some Inductive operations

P_2-to-C_4 Operation



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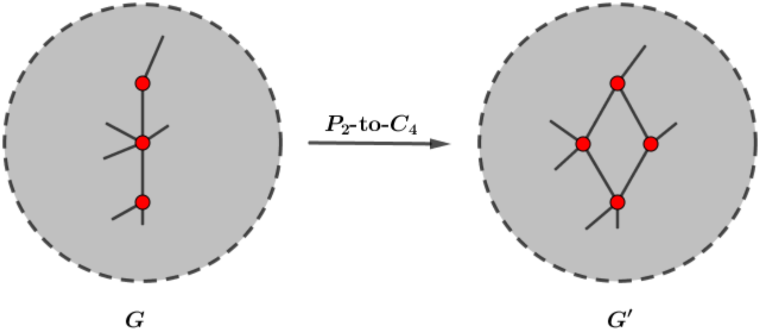
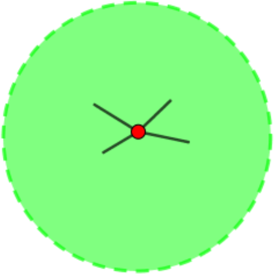
P_2-to-C_4 Operation



v-to-K_4 Operation



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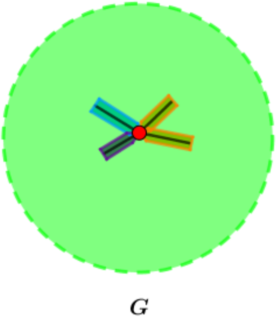

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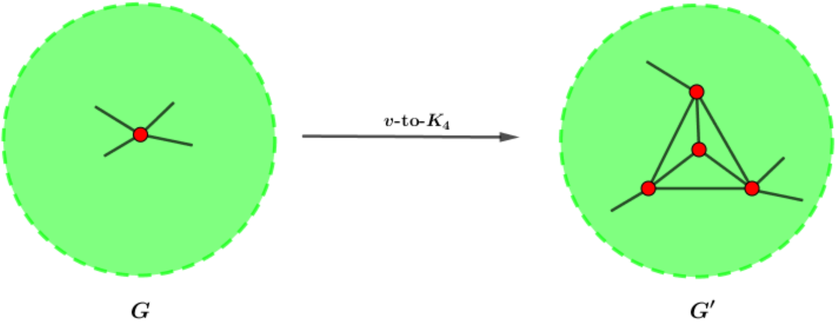

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- A graph G is a plane Laman ($(2, 3)$-tight) graph if and only if it can be obtained from an edge by plane vertex splitting operations.

References

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3. C. S. A. Nash-Williams. Edge-disjoint spanning trees of finite graphs. Journal London Mathematical Society, 36:445–450, (1961).