Instabilities in Soft Dielectric Plates

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Soft Dielectrics

Soft dielectric materials are smart materials that deform elastically in the presence of an electric field.

These materials can be used to produce actuators, artificial muscles or wearable electronics.

They are modelled by coupling the equations of electrostatics with those of non-linear elasticity.

Instabilities

Instabilities in soft dielectrics can cause material breakdown.

These instabilities can also be exploited for some applications.

The **snap-through** instability can be used to generate a large deformation, if it occurs before the material breaks down.

In practice, it is difficult to achieve this. The material reaches electrical breakdown before the snap-through can occur.

Instabilities

Other possible instabilities include **wrinkling** or thinning.

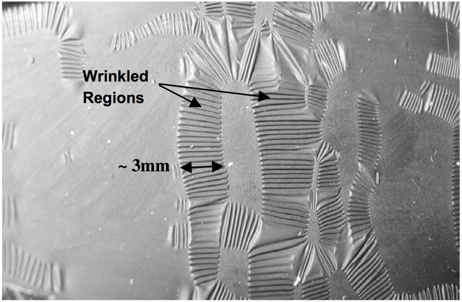


Figure: Experiment showing wrinkling instability in VHB4905/4910 [2]

Sinusoidal wrinkles form in soft dielectric plates under high voltage.

Electroelastic Deformations

Using the theory outlined by Dorfmann and Ogden [1], we construct the **Lagrangian** electric field and displacement,

$$\boldsymbol{E}_L = \boldsymbol{F}^\top \boldsymbol{E} \qquad\qquad \boldsymbol{D}_L = J\boldsymbol{F}^{-1}\boldsymbol{D}$$

where $\boldsymbol{F}$ is the deformation gradient, $J = \det \boldsymbol{F}$, and $\boldsymbol{E}$ and $\boldsymbol{D}$ are the electric field and displacement in the deformed configuration.

We choose $\boldsymbol{E}_L$ as our dependent electric variable and define the **total energy density function**, $\Omega = \Omega(\boldsymbol{F}, \boldsymbol{E}_L)$.

Electroelastic Deformations

If the material is **isotropic** and **incompressible** then Ω is a function of two invariants of the right Cauchy-Green deformation tensor $\boldsymbol{c} = \boldsymbol{F}^\top \boldsymbol{F}$, and three independent invariants of $\boldsymbol{E}_L$, [1].

$$I_1 = \operatorname{tr} \boldsymbol{c} \qquad I_2 = \frac{1}{2}[I_1^2 - \operatorname{tr}(\boldsymbol{c}^2)]$$

$$I_4 = \boldsymbol{E}_L \cdot \boldsymbol{E}_L \qquad I_5 = \boldsymbol{E}_L \cdot (\boldsymbol{c}^{-1}\boldsymbol{E}_L) \qquad I_6 = \boldsymbol{E}_L \cdot (\boldsymbol{c}^{-2}\boldsymbol{E}_L)$$

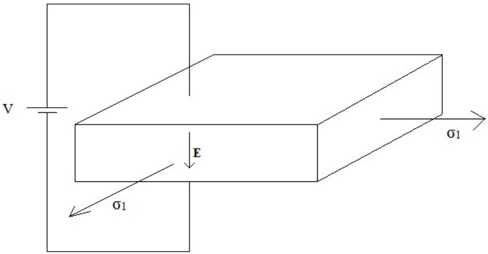
We construct the **total Cauchy stress tensor**

$$\boldsymbol{\tau} = \boldsymbol{F}\frac{\partial \Omega}{\partial \boldsymbol{F}} - p\boldsymbol{I} \qquad (1)$$

where p is a Lagrange multiplier introduced to take into account the incompressibility condition and Ω is now a function of the invariants.

Setup of Model

We consider a rectangular plate of soft dielectric material that is **stretched equally** along its lateral directions. We denote the principal stretches by $\lambda_1 = \lambda_2 = \lambda$, $\lambda_3 = \lambda^{-2}$.



We apply a voltage across the thickness direction so that the electric field is $\mathbf{E} = (0, 0, E_3)$, where E_3 is a constant. Then $\boldsymbol{E}_L = (0, 0, E_{L3})$, where $E_{L3} = \lambda^{-2} E_3$.

Incremental Deformation

We then superpose an **incremental deformation** onto this deformation and solve the incremental problem,

$$\operatorname{div} \dot{\boldsymbol{T}}_0 = \boldsymbol{0} \qquad \operatorname{div} \dot{\boldsymbol{D}}_{L0} = 0 \tag{2}$$

where $\dot{\boldsymbol{T}}_0$ and $\dot{\boldsymbol{D}}_{L0}$ are the push-forward versions of the increments of $\boldsymbol{T} = \boldsymbol{F}^{-1}\boldsymbol{\tau}$ and $\boldsymbol{D}_L$, respectively.

We linearise the equations and solve the incremental boundary problem using the **Stroh formulation**.

Thin Plate Equations

We find the equations for the thin plate for the **ideal dielectric** [4]

$$\Omega = \frac{\mu}{2}(I_1 - 3) - \frac{\varepsilon}{2} I_5 \tag{3}$$

and using the Stroh formulation we find the following equations for the thin plate,

$$\lambda^8 V^2 - \lambda^6 + 1 = 0 \tag{4}$$

for the antisymmetric mode, and

$$\lambda^8 V^2 - \lambda^6 - 3 = 0 \tag{5}$$

for the symmetric mode, where $V = \lambda^2 \sqrt{\varepsilon/\mu} E_3$ is a dimensionless measure of the voltage.

Thin Plate

A more complicated model such as the **Gent** is needed, in order for snap-through to occur.

However, we can compare the results to the purely elastic case and to existing models.

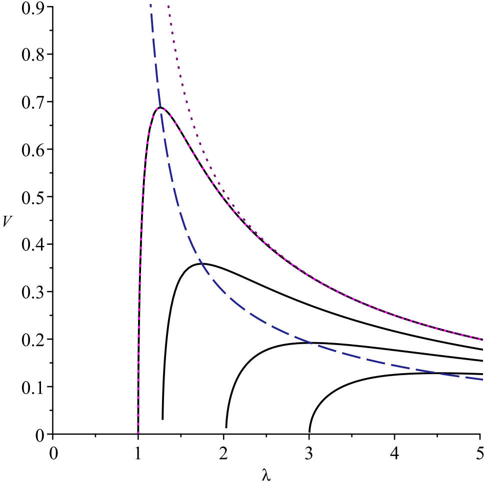


Figure: Thin plate instability

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