Noise Induced Transitions in the Nordmark Square Root Map

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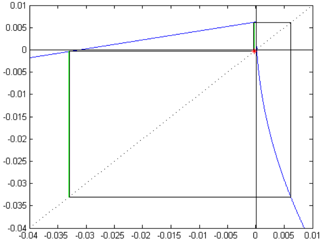
Nordmark's Square Root Map

Many impacting systems are described by a one-dimensional map known as the Nordmark square root map. The map is derived as an approximation for solutions of piecewise smooth differential equations near certain types of grazing bifurcation.



A grazing bifurcation is an effect of the fact that an impact with low velocity is sensitive to small changes in the initial conditions, with sensitivity inversely proportional to impact velocity.

$$x_{n+1} = S(x_n) = \begin{cases} \mu + bx_n & \text{if } x_n < 0 \\ \mu - a\sqrt{x_n} & \text{if } x_n \geq 0 \end{cases} \quad (1)$$

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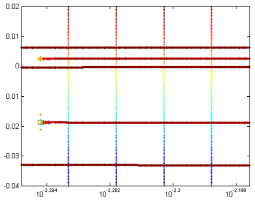
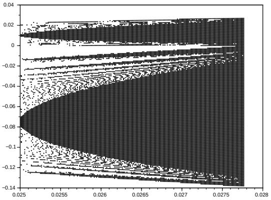
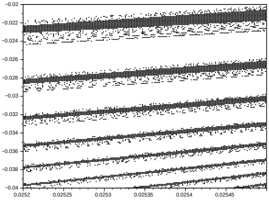
Multistability In the Square Root Map

If $0 < b < \frac{1}{4}$ there are values of $\mu > 0$ for which a stable periodic orbit with code $(RL^n)^\infty$ exists for each $n = 1, 2, \ldots$, and also such that there are two stable periodic orbits, one with code $(RL^n)^\infty$ and the other with code $(RL^{n+1})^\infty$.

For b in this range these are the only possible attractors except at bifurcation points.







Types of Noise

Simpson and Kuske make a careful analysis of how noise in impacting systems manifests in the map. They conclude that there are three different models:

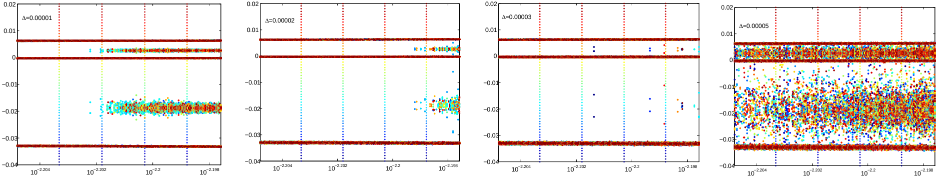
1. if the noise is in the impact itself then the constant a is replaced (in the limit of small noise) by $a(1+\frac{1}{2}\xi_k)$, i.e. the effect of this is parametric noise;
2. if there is noise in the position of the impacting surface then the switch at $x=0$ is replaced by a sum $x_k + A\xi_k = 0$ and this variable is used in the square root for the impact too.
3. In both of the above they consider coloured noise, the third case they consider is the small correlation time limit i.e. white noise on the impacting dynamics.

In an earlier paper Simpson, Hogan and Kuske consider additive Gaussian noise of amplitude, Δ, and show that this particular noise formulation arises in a general setting.

The Effect of Noise

My work thus far has focused on phase space sensitivity for period two and three coexistence, investigating a shift of the proportion of points going to one behaviour or the other, for both parametric and additive noise.

The results have not been entirely as we had expected. The relationship between the proportion of points going to each of the coexisting attractors is not monotonic.



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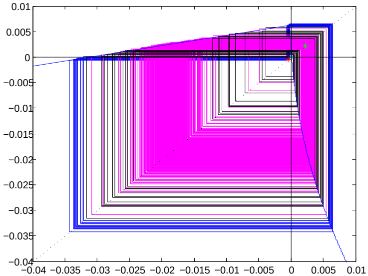
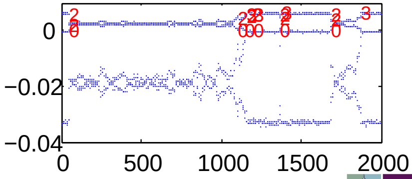
The Transition Mechanism

Perhaps the most interesting phenomenon that we have observed is the potential for repeated intervals of persistent RL dynamics in a noisy system with $\mu < \mu_2^s$.

In the case of both additive and parametric noise, we have observed that the noise-induced transition between RLL and RL behaviour in this case takes the following symbolic form

$$RLLRLL\ldots RLL\underline{RLLRLRRL}RL\ldots RL. \tag{2}$$





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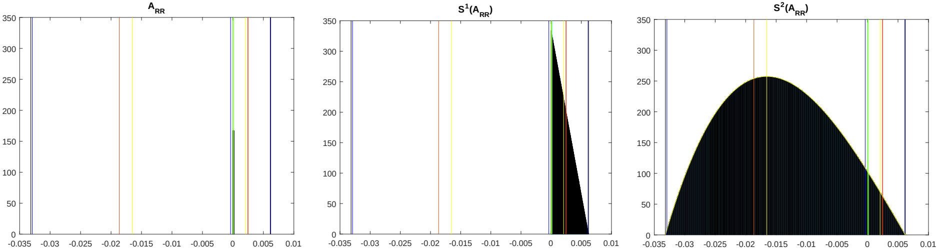
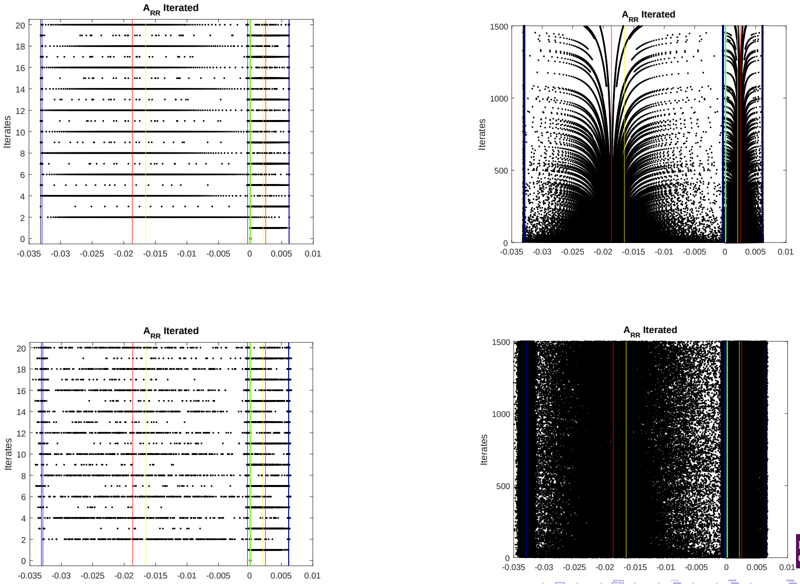
The Transition Mechanism

The most significant feature of the transition given in (2) is the repeated R, corresponding to repeated low velocity impacts.



These repeated low velocity impacts allow the dynamics to be pushed into the region of phase space with slow dynamics, in the vicinity of the unstable $(RL)^\infty$ orbit of the deterministic system.

The Transition Mechanism



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Future Work

- We would like to get a better idea of what conditions are required on the noise to induce a repeated low velocity impact during RLL behaviour and hence a transition to RL behaviour.
- We would like to investigate what conditions are required on the noise to maintain RL behaviour for a significant number of iterates after the transition.
- How does this problem scale in relation to noise amplitude as we look at regions of RL^n and RL^{n+1} coexistence for increasing n.

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References

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