Noise, Deviation Distributions and Multistability in the Square Root Map

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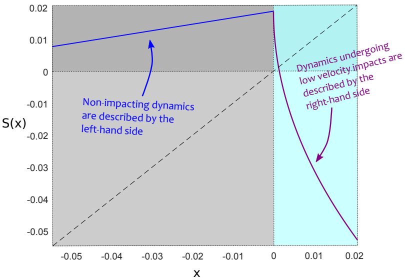
The Square Root Map

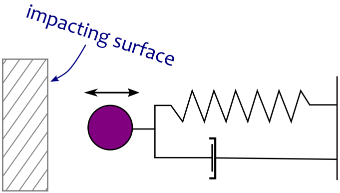
Many physical real-world systems can be described as impacting and are modelled using impact oscillators.

Near low-velocity impacts, impact oscillators can be described by a one-dimensional map known as the square root map.



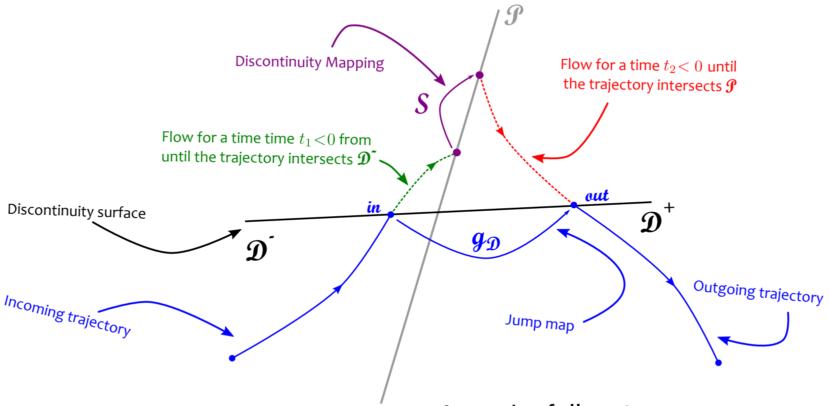
The Square Root Map

$$x_{n+1} = S(x_n) = \begin{cases} \mu + b x_n & \text{if } x_n < 0 \\ \mu - a\sqrt{x_n} & \text{if } x_n \geq 0 \end{cases} \tag{1}$$



A forced impact oscillator

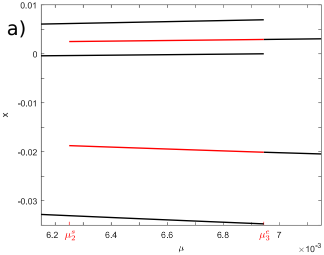
The Square Root Map

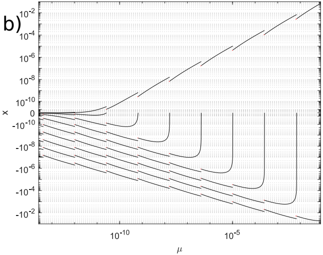


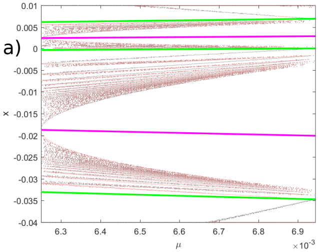
Deriving the map from the full system

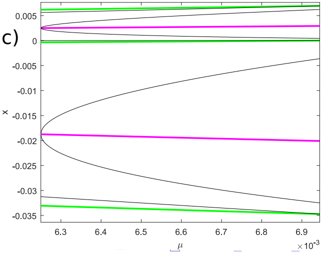
Multistability In the Square Root Map

For $0 < b < \frac{1}{4}$ the square root map displays a bifurcation structure known as a *period-adding cascade*.









The Square Root Map With Additive Noise

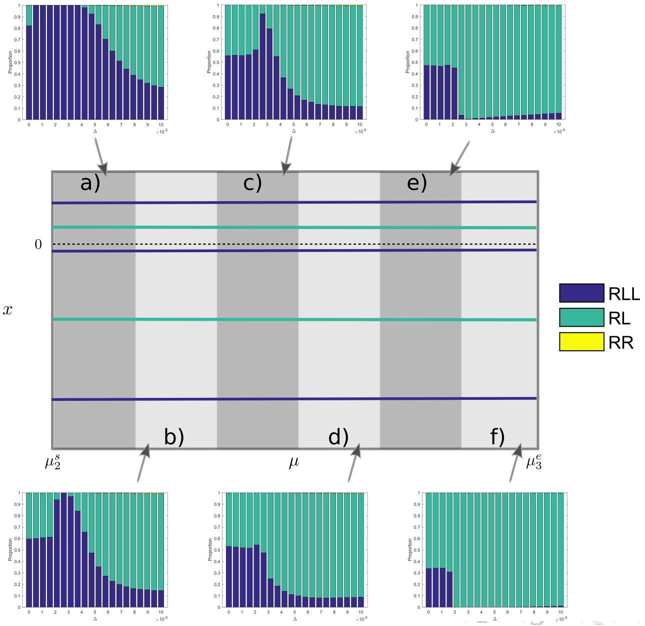
In [SHK13] Simpson, Hogan and Kuske show that white noise in the piecewise smooth flow translates to additive white noise in the square root map. This noise formulation may be sensible to model systems where the forcing term or external fluctuations represent a significant source of uncertainty.

The square root map with additive Gaussian white noise is given by

$$x_{n+1} = S_a(x_n) = \begin{cases} \mu + bx_n + \xi_n & \text{if } x_n < 0 \\ \mu - a\sqrt{x_n} + \xi_n & \text{if } x_n \geq 0, \end{cases} \tag{2}$$

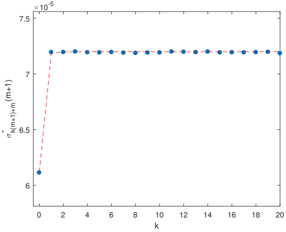
where ξ_n are identically distributed independent normal random variables with mean 0 and standard deviation Δ, $\xi_n \sim N(0, \Delta^2)$.

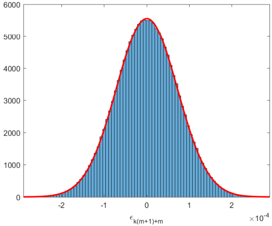
Proportions

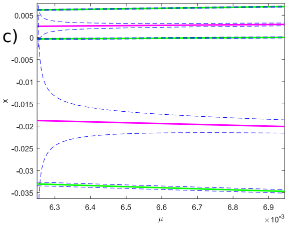


Approximating Trajectory Deviations

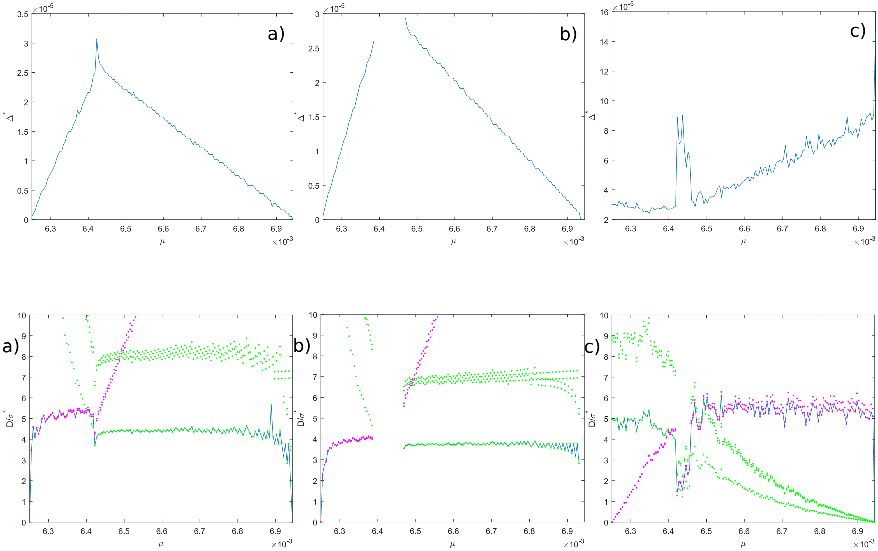
We consider two trajectories $\{x_k\}$ and $\{z_k\}$ with identical initial conditions x_0 and z_0 equal to the right iterate of the deterministic period-$(m+1)$ orbit of the system. We then iterate forward using the deterministic square root map in the case of z_0, (1), (2), and the square root map with additive noise in the case of x_0. The deviation due to noise in the trajectory $\{x_k\}$ is then given by the difference $\{\epsilon_k\} = \{x_k - z_k\}$.







Deviations as Predictors



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