

Dynamical System End of The Year Blitz Talk

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Consider a Network of N dynamical systems (agents) linearly connected:

$$\dot{x}_i = -x_i^2 + b + \sigma \sum_{j=1}^N a_{ij}(x_j - x_i), \quad \text{for } i = 1 \dots N \quad (1)$$

a_{ij} : adjacency matrix (undirected, $a_{ij} = a_{ji}$). And $\sigma \in \mathbb{R}$.

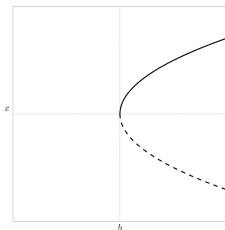
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Saddle-node: $f(x) = -x^2 + b$

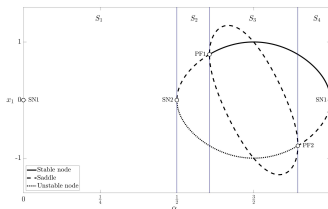
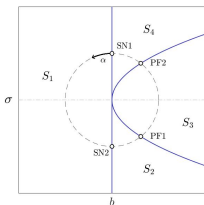


2. Region of reduced dynamics (R.R.D.)

$$\begin{aligned}\dot{x}_1 &= -x_1^2 + b + \sigma(x_2 - x_1), \\ \dot{x}_2 &= -x_2^2 + b + \sigma(x_1 - x_2),\end{aligned}\tag{2}$$

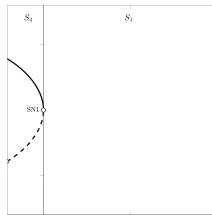
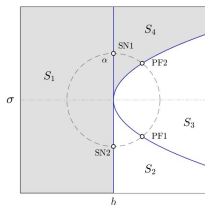
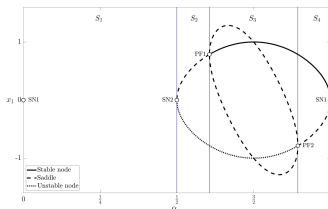
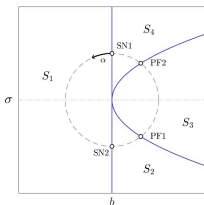
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3. Does the R.R.D. exist for any N ?

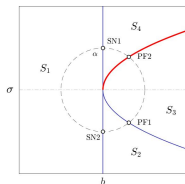
3. Does the R.R.D. exist for any N ?

Conjecture

Consider

a network of agents with saddle-node structure.
Then, for any connected topology and N , $b > 0$,
the lower bound for σ of the R.R.D. is given by

$$\sigma_c(b, N) \leq \sigma(b, N) \leq \sigma_p(b, N) \quad (3)$$



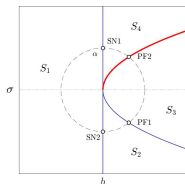
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- $\sigma_c(b, N)$ R.R.D. lower bound *complete* network
- $\sigma_p(b, N)$ R.R.D. lower bound *path* network

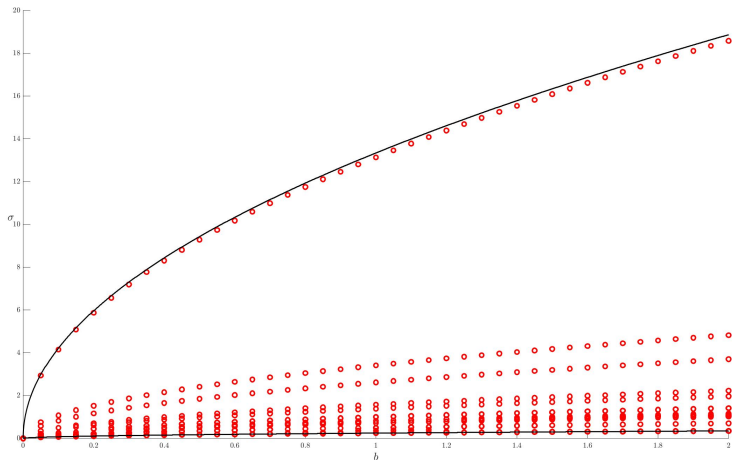
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5. Further work

- Paper publishing
- Prove conjectures
- Work on Control (Paper with Davide Di Palo)
- Collaborations and Applications