Decision space visualizations of mouse-tracking data

Using movement information to characterize dynamic decision processes

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Introduction

Goals

There are four main goals for this research:

1. Development of a methodology that represent the cognitive process during individual's binary choice (i.e. decision spaces).

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3. Control using bistable systems techniques
4. Online tool development

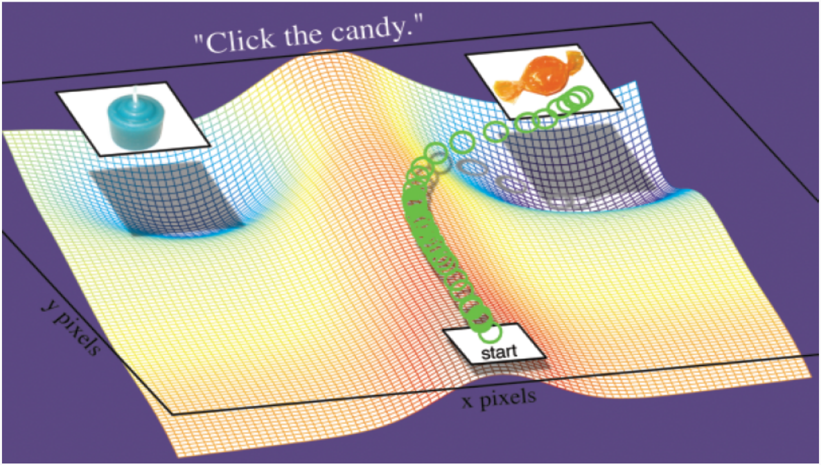
Decision Spaces

Brief History

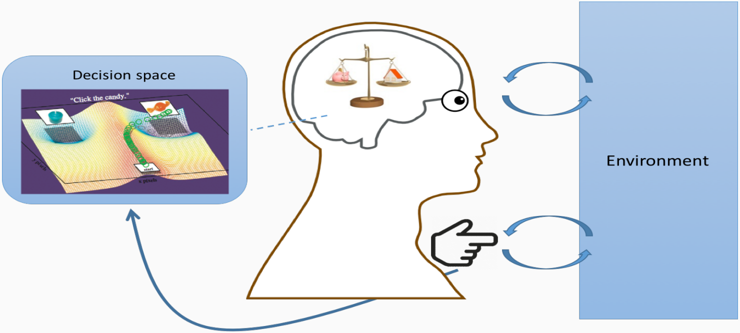
The analysis of DM changes along the years focusing of different aspects of the decisional process. Experimental psychologists in the area of decision making have been mainly interested:

- **What** people choose (1960) $\rightarrow$ **outcomes**, (e.g. Expected Utility Theory)
- **How long** it takes for them to arrive to the final decision under different circumstances. (1970-2000) $\rightarrow$ **response times**
- Information on **motor execution** of a decision(2000-2010) $\rightarrow$ **mouse trajectories**

Spivey and Dale theory

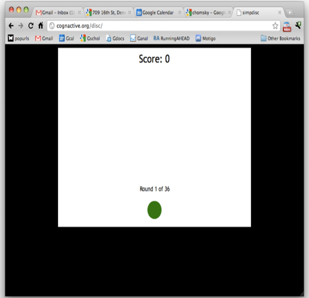


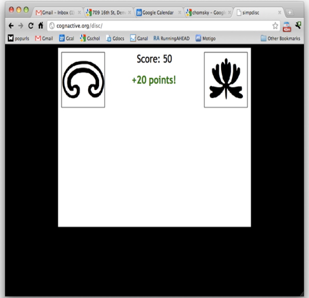
Spivey, M. J., Dale, R. (2006). Continuous Dynamics in Real-Time Cognition. Current Directions in Psychological Science, 15(5), 207-211.



Spivey argued that human mind smoothly travels in the continuous space of all possible mental states, rather than hopping between discrete representations.The landscape is what we define a decision space and is an expression of the cognitive evaluation of the available outcomes and the motor execution of the response influenced by this evaluation.

O’Hora experiment





Three different choices among:

- 7/5
- 20/5
- 10/5

Three kind of experiments:

- high/high
- low/low
- low/high

Data processing

Original data set:

- subject id
- trial number
- x co-ordinates
- y co-ordinates
- time grid
- options available (low/low, high/low, high/high)
- option chosen

Processed data set:

- Normalization of the time grid: setting of actual start time for each trajectory to 0.

O'Hora, D., Dale, R., Piiroinen, P. T., & Connolly, F. (2013). Local dynamics in decision making: The evolution of preference within and across decisions. Scientific reports, 3.

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- Averaging of the starting point of all the trajectories moving the origin from the top left of the screen to this point.

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Data processing

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Processed data set:

- Normalization of the time grid: setting of actual start time for each trajectory to 0.
- Averaging of the starting point of all the trajectories moving the origin from the top left of the screen to this point.
- Normalization of the direction of the trajectories: high choices set to the right, low choices set to the left

O'Hora, D., Dale, R., Piiroinen, P. T., & Connolly, F. (2013). Local dynamics in decision making: The evolution of preference within and across decisions. Scientific reports, 3.

Mathematical model

Using the computer-mouse coordinates collected during choices, it was assumed that motion during a choice can be described by a potential field given by the function $V(x, y) = V(x) + V(y)$, where x and y are the screen coordinates. At first an approximation was required, assuming that the motion in the x-direction was independent of motion in y-direction, therefore the potential could be separated in tho different functions: $V_x(x)$ and $V_y(y)$.

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The following system of differential equations, for a type of damped oscillators, models the system:

$$\frac{d^2x(t)}{dt^2} + d_1\frac{dx(t)}{dt} = -\frac{dV_x(x)}{dx}$$
$$\frac{d^2y(t)}{dt^2} + d_2\frac{dy(t)}{dt} = -\frac{dV_y(y)}{dy}$$

Velocity and acceleration

Finite difference method was used to find the velocity and acceleration in both x and y direction for each time step. Due to the different length of each time interval it was required to define two different time intervals: $h_1 = t_{i+1} - t_i$ and $h_2 = t_i - t_{i-1}$.

So:

$$\left(\frac{dx}{dt}\right)_i \simeq \frac{x_{i+1} - x_{i-1}}{h_1 + h_2}$$

$$\left(\frac{d^2x}{dt^2}\right)_i \simeq \frac{2(h_2 x_{i+1} - (h_1 + h_2)x_i + h_1 x_{i-1})}{(h_1 + h_2)h_1 h_2}$$

Velocity and acceleration

where:

$$x_{i+1} = x_i + h_1 x_i^{'} + \frac{h_1^2 x_i^{''}}{2!} + \frac{h_1^3 x_i^{'''}}{3!} + ...$$
$$x_{i-1} = x_i - h_2 x_i^{'} + \frac{h_2^2 x_i^{''}}{2!} - \frac{h_2^3 x_i^{'''}}{3!} + ...$$

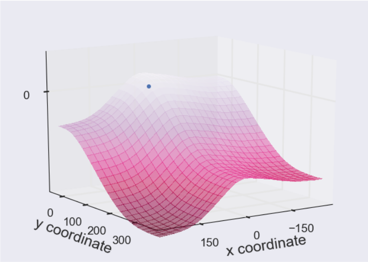
and:

$$h_2 x_{i+1} + h_1 x_{i-1} = x_i(h_1 + h_2) + \frac{1}{2} h_1 h_2 (h_1 + h_2) x_i^{''} + \frac{1}{6} h_1 h_2 (h_1^2 + h_2^2) x_i^{'''} + ...$$

Row Wise interpolation

Original work was performed with a *Row-Wise* technique:

- Screen divided in several rows.
- Velocity and acceleration obtained with interpolation and average at each mesh point.
- Calculation of the function V via the integrals:



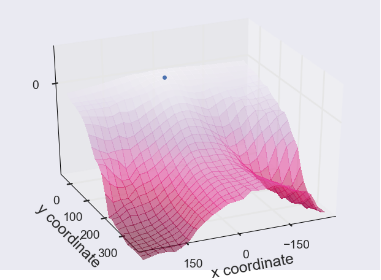
$$
\begin{aligned}
V_x(x) &= -\int_x \frac{d^2\hat{x}(t)}{dt^2} + \frac{d\hat{x}(t)}{dt}\,dx \\
V_y(y) &= -\int_y \frac{d^2\hat{y}(t)}{dt^2} + \frac{d\hat{y}(t)}{dt}\,dy
\end{aligned}
$$

Cell Wise interpolation

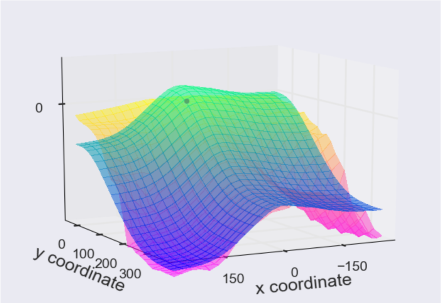
New work is performed with a *Cell-Wise* technique:

- Screen divided in several cells.
- Velocity and acceleration obtained with interpolation and average at each mesh point.
- Calculation of the function V via the integral:

$$V(x, y) = \int_{(0,0)}^{(x,y)} (\dot{x}+\ddot{x}, \dot{y}+\ddot{y})\, dr$$



Cell-Wise vs. Row Wise



It's clear to notice that taking into account the dependence between the two directions we are able to produce a much better result

Identification

Mathematical Model

Our idea is that the potential function is a polynomial model such as:

$$V(x,y) = V_x(x) + V_y(y) + V_{xy}(x,y),$$

$$V_x(x) = \int \frac{\partial V_x}{\partial x} = \int x(x+1)(x-1) = \frac{x^4}{4} - \frac{x^2}{2},$$

$$V_y(y) = \int \frac{\partial V_y}{\partial y} = \int y(y-1) = \frac{y^3}{3} - \frac{y^2}{2},$$

$$V_{xy}(x,y) = c_{11}xy + \frac{c_{21}}{2}x^2y + \frac{c_{12}}{2}xy^2 + \frac{c_{31}}{3}x^3y + \frac{c_{22}}{2}x^2y^2 + \frac{c_{13}}{3}xy^3.$$

Mathematical Model

Where V is the potential function of this system:

$$\begin{aligned}\tau_x \dot{x} &= -\frac{\partial V}{\partial x}, \\ \tau_y \dot{y} &= -\frac{\partial V}{\partial y},\end{aligned}$$

Deriving potential equations we can obtain the dynamical system :

$$\tau_x \dot{x} = -(x^3 - x) + c_{11} y + c_{21} xy + c_{31} x^2 y + c_{22} xy^2 + \frac{c_{12}}{2} y^2 + \frac{c_{13}}{3} y^3$$
$$\tau_y \dot{y} = -y(y-1) + c_{11} x + \frac{c_{21}}{2} x^2 + \frac{c_{31}}{3} x^3 + c_{22} x^2 y + c_{12} xy + c_{13} xy^2$$

Parameters identification

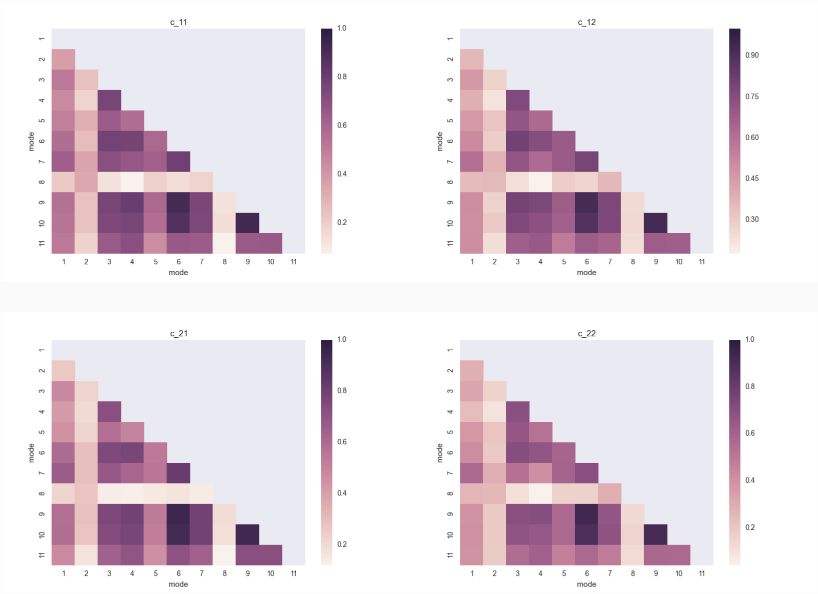
- Minimization of the error between the experimental velocity and the model.
- Minimization for initial conditions and time grid
- Error function expressed in term of ordinary least square sum.
- Performance analysis on all trajectories filtered by x-flips
- 13 different minimization methods

1. Powell
2. Nelder-Mead
3. BFGS
4. CG
5. Newton-CG
6. L-BFGS-B
7. TNC
8. COBYLA
9. SLSQP
10. Differential Evolution
11. Basing Hopping Powell
12. Basing Hopping N-Mead
13. Basing Hopping L-BFGS-B

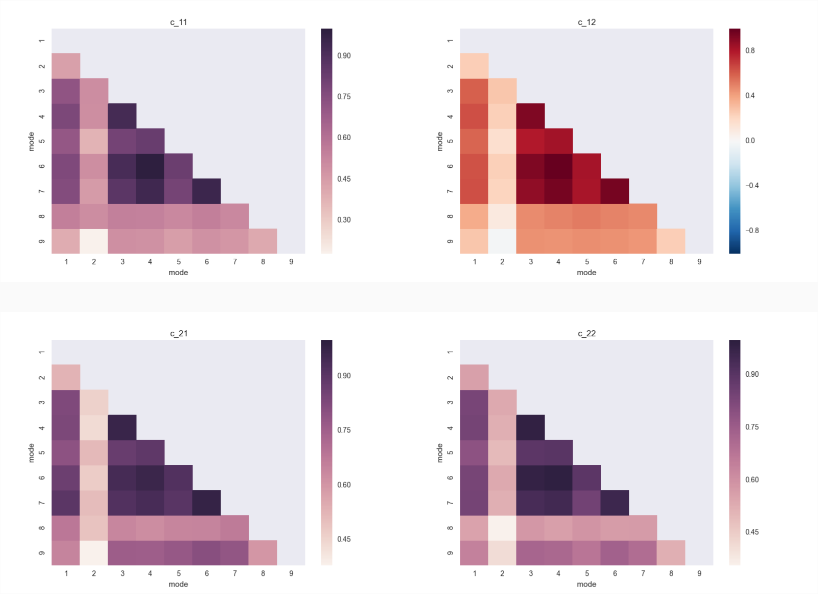
Parameters identification performance analysis

ID	Method	Type (default Local, unconstrained)	Median error	Median nfev	std(c_11)
1	Powell		68	1761	1,34E+24
2	Nelder-Mead		110	1159	9,65E+09
3	CG	Jacobian	62	1901	47
4	BFGS	Jacobian	55	84	2E+10
5	Newton-CG	Jacobian	80	45	28,39
6	L-BFGS-B	Constrained, Jacobian	58	133	0,85
7	TNC	Constrained, Jacobian	64	100	0,90
8	COBYLA		150	1000	0,44
9	SLSQP	Constrained, Jacobian	58	52	0,79
10	Differential Evolution	Global, Constrained	58	121047	0,88
11	Bas. Hopping Powell	Global	56	198189	1,34E+24

Spearman's rank correlation by trial



Spearman's rank correlation by subject

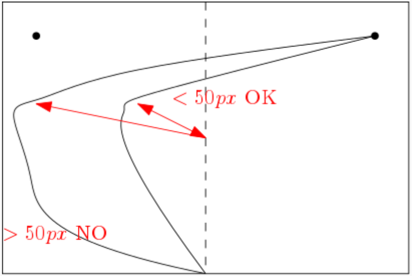


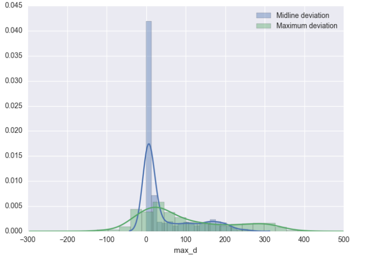
Analysis

Data filtering

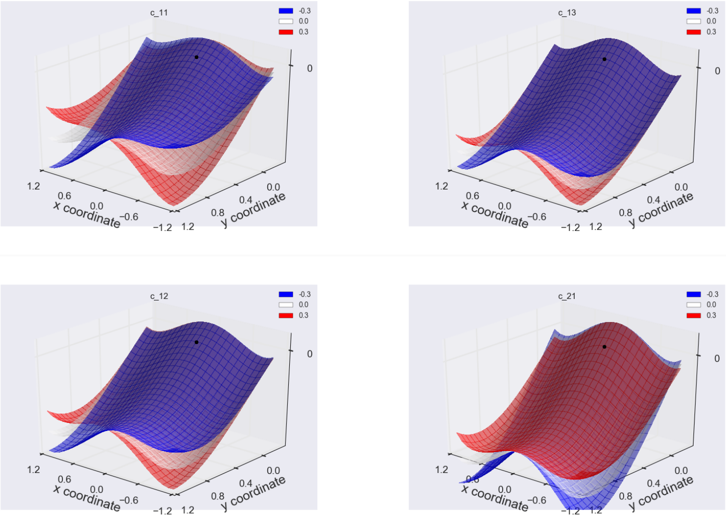
The trajectories have been filtered out following those criterias:

- Change of mind parameter: max. deviation $> 50px$
- Parameters result: True/False optimization routine
- Number of x-flips: x-flip<2

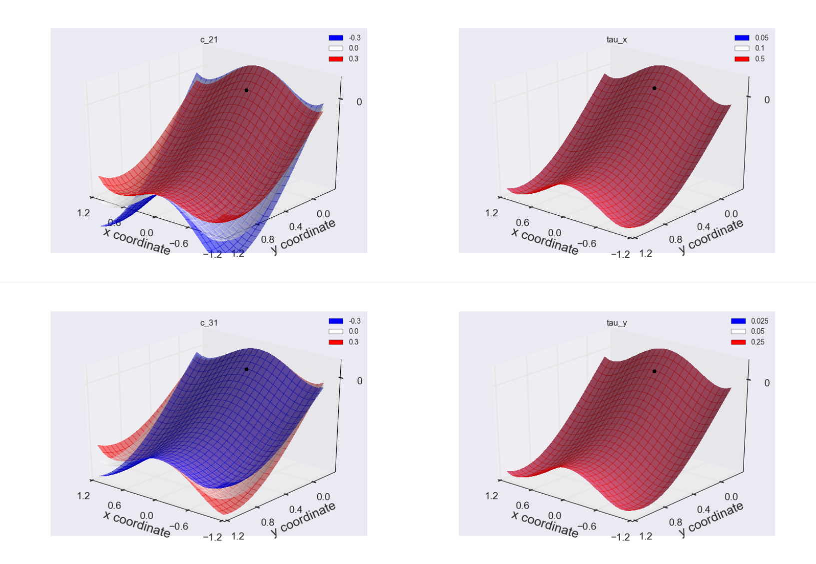




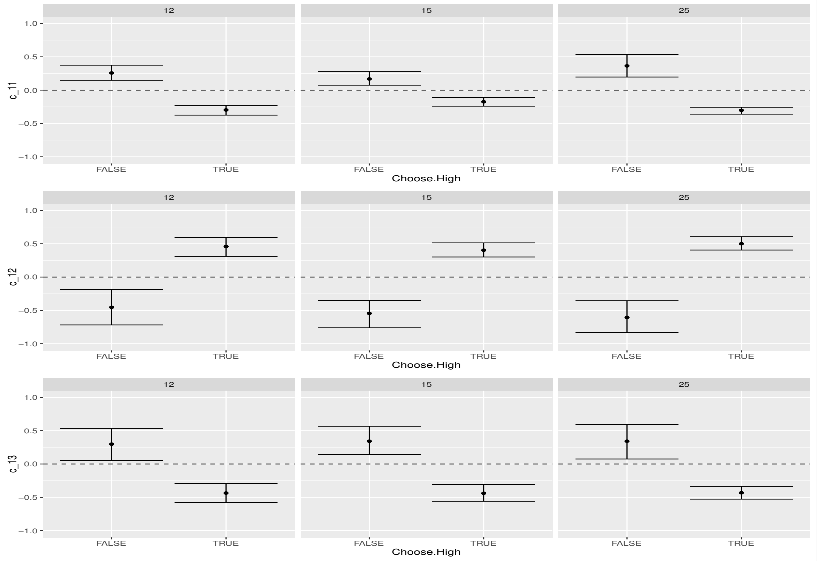
Study of parameters I



Study of parameters II



Study of parameters III

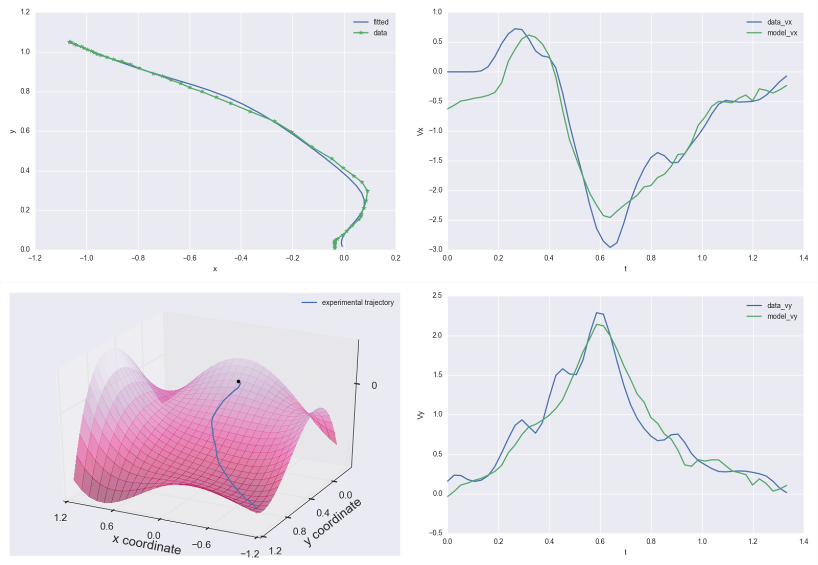


Single trajectory analysis

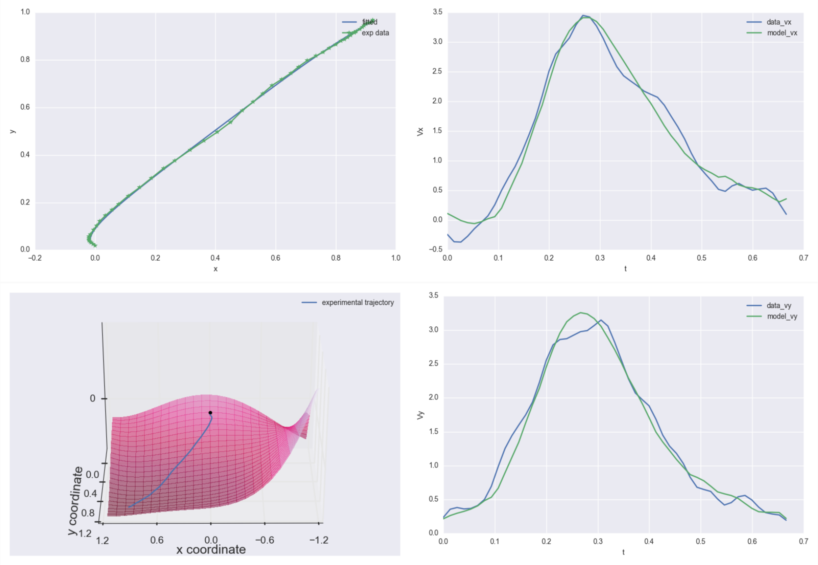
The single trajectory analysis is divided in three parts:

- Parameters identification: 1889 trajectories for 126 individuals, 51 data points each.
- Initial conditions and time grid identification using bruteforce
- Model simulation

Subject 8972: trial 3



Subject 5582: trial 36

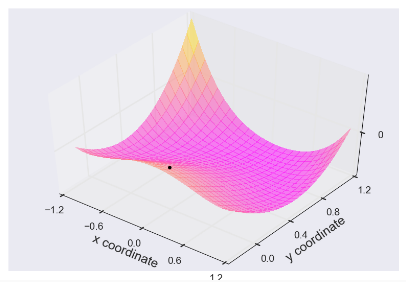


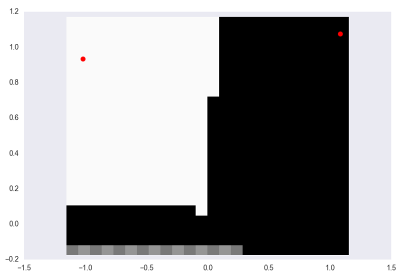
Multi trajectory analysis

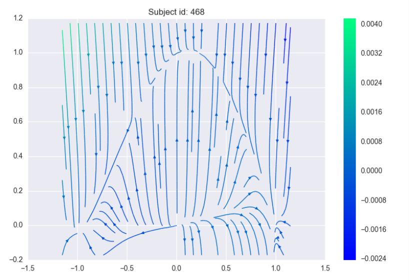
The identification procedure for a multi trajectory system has been performed minimizing the error function across all the trials of a single individual.

- 126 individual surfaces.
- 12 minimization procedures using the previous methods
- Quiver plot
- Basin of attraction analysis

Subject id: 468

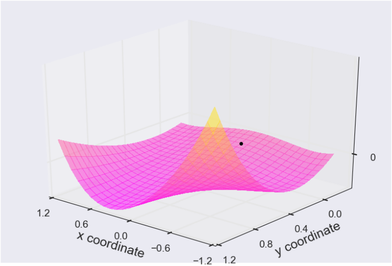


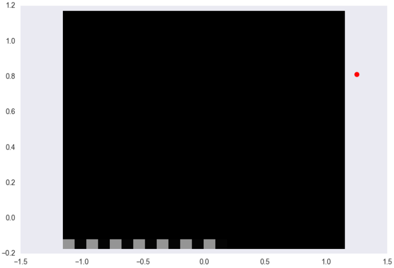


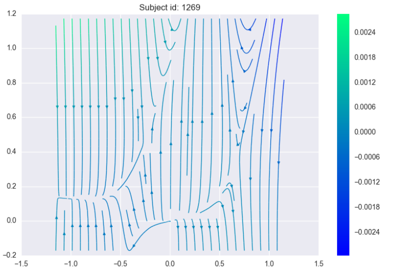


- Minimization method: L-BFGS-B
- Error: 811.42
- Nfev: 86

Subject id: 1269







- Minimization method: L-BFGS-B
- Error: 1616
- Nfev: 106

Control

Introduction

It's possible to divide the control problem in two macro categories:

- Single agent control
- Multi agent control

With single control we mean a controller that operates with an external input on a single trajectory (or individual behavior) trying to drift the system from one point of equilibrium to the other.
The multi agent control problem, instead, studies how the other individuals could influence each others.

Single agent control

Single agent control is performed on a single trajectory drifting the response from the left (*wrong choice*) to the right (*right choice*)

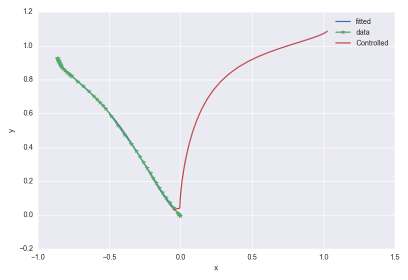
The idea is to introduce a control action u whether one of the these two situation occurs:

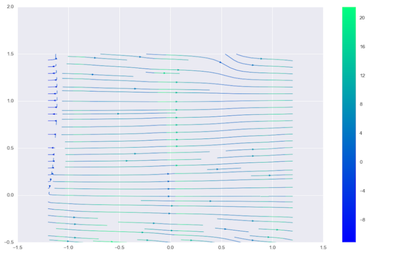
- $\dot{x} < 0$ and $t > \alpha$
- $x < 0$

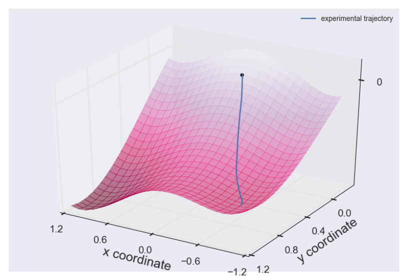
where $u = \gamma > |f(x, y)|$ is applied to the system:

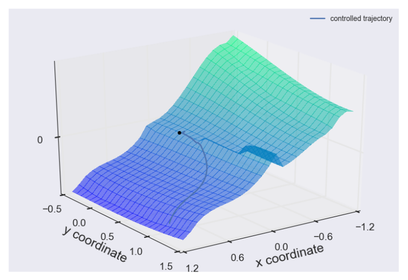
$$\dot{x} = f(x, y) + u$$
$$\dot{y} = g(x, y)$$

Velocity feedback control (id 223 trial 19





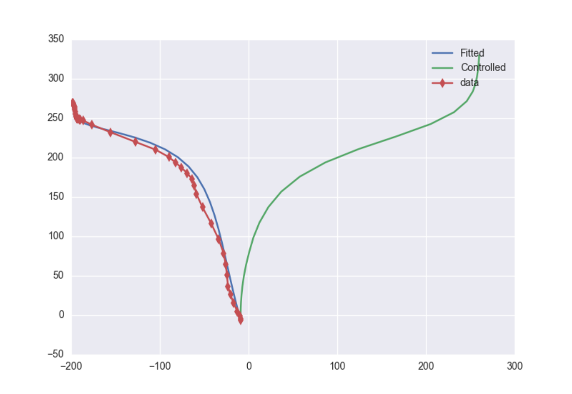




Position feedback control

Subject id: 6634

Trial: 12



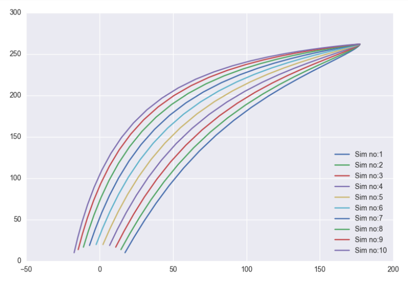
Multi agent control

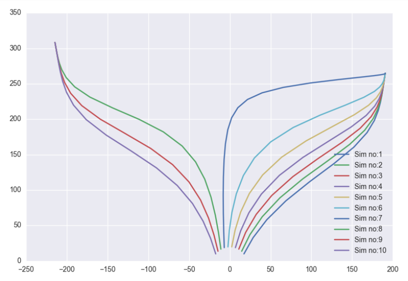
The multi agent control is developed connecting two or more system among each other. In this experiment we suppose that individual no. 1 influence individual no. 2.

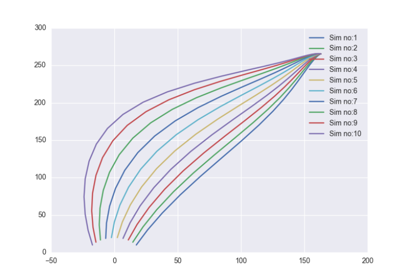
The equations that describe the control system are:

$$\begin{aligned}
\dot{x}_1 &= f_1(x, y) \\
\dot{y}_1 &= g_1(x, y) \\
\dot{x}_2 &= f_2(x, y) + \sigma(\dot{x}_1 - \dot{x}_2) + \gamma(x_1 - x_2) \\
\dot{y}_2 &= g_2(x, y) + \sigma(\dot{y}_1 - \dot{y}_2) + \gamma(y_1 - y_2)
\end{aligned}$$

Multi agent control II







Not controlled id: 8707

Controlled id: 7996

$\sigma = 0.01$, $\gamma = 0.015$

Questions?

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