Torsion units in integral group rings of sporadic simple groups

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Let $V(\mathbb{Z}G)$ be the normalized unit group of the integral group ring $\mathbb{Z}G$ of a finite group G. The long-standing conjecture of H. Zassenhaus **(ZC)** says that every torsion unit $u \in V(\mathbb{Z}G)$ is conjugate within the rational group algebra $\mathbb{Q}G$ to an element in G.

W. Kimmerle proposed to relate **(ZC)** with some properties of graphs associated with groups. The Gruenberg–Kegel graph (or the prime graph) of G is the graph with vertices labelled by the prime divisors of the order of G with an edge from p to q if and only if there is an element of order pq in the group G. Then Kimmerle's conjecture **(KC)** asks whether G and $V(\mathbb{Z}G)$ have the same prime graph.

We started the program of verifying **(KC)** for sporadic simple groups, using the Luthar–Passi method with recent extensions by M. Hertweck as a main tool. Currently we are able to report on the checking **(KC)** for the following twelve groups (see [1–5] and upcoming papers at `http://arxiv.org/`):
– Mathieu sporadic groups M_{11}, M_{12}, M_{22}, M_{23}, M_{24};
– Janko sporadic groups J_1, J_2, J_3;
– Higman-Sims sporadic group HS, McLaughlin sporadic group McL, Rudvalis sporadic group Ru and Suzuki sporadic group Suz.

References

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