# Linear Algebra and Matrix Theory

## connections, applications and computations

6$^{th}$ de Brún Workshop

National University of Ireland, Galway
December 3–7, 2012.
http://www.maths.nuigalway.ie/deBrun6

## Contents



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Hosted by de Brún Centre, NUI Galway,
and supported by the Science Foundation Ireland.

# 1 Programme and General Information

Welcome the the 6th de Brún Workshop and to Galway.

All talks (Monday to Friday) will take place in rooms IT250 or IT125, Information Technology Building, on the main campus of NUI Galway. Talks in the Irish Meeting for Linear Algebra Research on Saturday will take place in Room AC201 on the main concourse in NUI Galway.

The de Brún Workshop is funded by Science Foundation Ireland whose support is gratefully appreciated.

## *Schedule*

| | **Monday** | **Tuesday** | **Wednesday** | **Thursday** | **Friday** |
|---|---|---|---|---|---|
| 9.00 | Registration and Coffee | | | | |
| 9.30 | C.M. da Fonseca | C.M. da Fonseca | I. Duff | C. Johnson | C. Johnson |
| 10.30 | P. Brooksbank | P. Brooksbank | I. Duff | C.M. da Fonseca | C.M. da Fonseca |
| 11.30 | Tea / Coffee | | | | |
| 12.00 | C. Johnson | C. Johnson | P. Brooksbank | R. Brualdi | R. Brualdi |
| 1.00 | Lunch | | | | |
| 2.30 | R. Levene | A. Bier | C. Arauz | J. Lee | J. Cruickshank |
| 3.00 | E. Byrne | J. Armario | W. Holubowski | M. Saha | A. Dmytryshyn |
| 3.30 | J. McTigue | T. Klimchuk | A. Cronin | O. Walch | A. Rahm |
| 4.00 | Tea / Coffee | | | | Close |
| 4.30 | L. Taslaman | R. Brualdi | I. Duff | T. Hurley | |
| 5.00 | R. Egan | | | I. McLoughlin | |

*Internet Access*: Participants can access the **nuigwifi** network using the following credentials :

*Username:* 9876259T *Password:* gvaxp2694

*Workshop Dinner*: The workshop dinner will take place at 8.00 on Wednedsay night, at the House Hotel which is located at Spanish Parade in the city centre (the location is marked on the map in the conference folder). The cost of the dinner is €30 per person. Please inform one of the local organisers on Monday if you plan to attend.

*Organizing Committee*:
Graham Ellis, NUI Galway (local organiser)
Stephen Kirkland, NUI Maynooth
Thomas Laffey, University College Dublin
Niall Madden, NUI Galway (local organiser)
Rachel Quinlan, NUI Galway (local organiser)
Helena Šmigoc, University College Dublin
If you have any queries, please contact one of the local organisers.

Thanks to Mary Kelly, Noelle Gannon, James McTigue, Stephen Russell and Nhan Anh Thai for additional organisational support.

# 2 Short Courses

### Linear Methods in Computational Algebra

**Peter Brooksbank**
Bucknell University, Pennsylvania

In this series of lectures I aim to provide an overview of some fundamental problems in computational algebra, focusing particularly on problems that admit the use of techniques from linear algebra. Not surprisingly, the selection of problems to some extent reflects my own interests, but each of them is an active area of current research in the field. I assume only a basic knowledge of algebra: groups, rings, fields, and of course linear algebra.

**Lecture 1.** In this lecture I will give a (somewhat biased) overview of computational algebra, putting a special emphasis on problems concerning linear groups and algebras. I will discuss broad goals, some history, algorithms, and complexity. I will also introduce fundamental tools needed for the subsequent talks.

**Lecture 2.** The basis for this lecture is a large scale project to compute the structure of any given group of matrices defined over a finite field. I will mention two approaches to this problem and identify a certain problem of "constructive recognition" as being crucial to both. I will then discuss algorithms to solve the latter problem, once again emphasizing the power of techniques from linear algebra.

**Lecture 3.** The motivation for this lecture is the fundamental "isomorphism problem" for finite groups, which seeks an efficient algorithm to decide whether or not two given finite groups are isomorphic. I will show how, in key cases, the problem involves the construction of groups that preserve a certain linear object (a bi-additive map) associated to the input groups. The main idea is to then "linearize" a seemingly non-linear problem; I will report on recent progress in this regard.

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### Topics in Combinatorial Matrix Theory

**Richard Brualdi**
University of Wisconsin–Madison

Topics to be taken from: Alternating sign matrices, Combinatorial Matrix Classes ((0,1)-matrices, doubly stochastic matrices and generalizations, ...), A Berlekamp switching game, Principal minors of symmetric matrices, ... .

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### Sparse Matrices

**Iain Duff**
Rutherford Appleton Laboratory, Oxfordshire, and CERFACS, Toulouse

This series of lectures will introduce and illustrate the ubiquity of sparse matrices and will discuss in some detail the solution of large sparse linear systems. We will mainly consider direct methods of solution based on a matrix factorization but also discuss iterative and hybrid methods. The only assumption made of the audience is that they are familiar with linear equations and matrices.

Good background reading might include the books:

- Direct Methods for Sparse Linear Systems. Timothy A. Davis. SIAM Press 2006.
- Direct Methods for Sparse Matrices. Iain S. Duff, Albert M. Erisman, and John K. Reid. Oxford University Press. 1986.
- Computer Solution of Large Sparse Positive Definite Systems. A. George and J. W. H. Liu. Prentice-Hall. 1981.

**Lecture 1.** Introduction to sparse matrices and sparse linear equation solution and introduction to sparse direct methods. This will include relationship to graphs and matrix reorderings.

**Lecture 2.** Direct methods for solving sparse linear systems to include discussion of elimination trees, supernodal and multifrontal methods and comments on parallelism.

**Lecture 3.** A short introduction to iterative and hybrid methods with a discussion of current research in this area.

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## Recent Trends on Spectral Graph Theory

**Carlos Martins da Fonseca**
Universidade de Coimbra, Portugal

Spectral graph theory is a prosperous and powerful common branch of graph theory and linear algebra that relates various graph properties with the spectra of certain matrices associated to the graph. This vibrant field of interest has been extensively studied in the last decades with applications in various disciplines.

The aim of these lectures is to recall some crucial results on spectral graph theory and understand some recent developments in the area.

In the first session we will review basic results on the spectra of acyclic Hermitian matrices and its relations with the underlying graph, reminding some well and less-known results.

The next two sessions will be devoted to the P-vertices of acyclic symmetric matrix matrices. We will characterise the trees where the maximum number of P-vertices is attained. Moreover, we will establish an algorithm to construct a graph and a corresponding matrix where the number of P-vertices is given. One session will be dedicated to the nonsingular matrices and the other to the remaining matrices.

In the last lecture, we will analyse the multiplicities of the eigenvalues of the so-called -binary tree. We carry this discussion forward extending some recent results to a larger family of trees, namely, the wide double path, i.e., a tree consisting of two paths that are joined by another path. Several research problems will be proposed.

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## Totally Positive Matrices

**Charles Johnson**
College of William and Mary, Virginia

General goal: The audience should be exposed a fairly complete description of all the basic theory, and have an idea of some recent work and interesting research problems in the subject.

**Lecture 1.** Definitions, history, common tools, and the most basic theory

**Lecture 2.** The Elementary Bidiagonal Factorization, applications, recognition results and spectral theory.

**Lecture 3.** Advanced Topics: determinantal inequalities, shadows, Hadamard powers, etc

**Lecture 4** Modern Research: completion problems, geometric connections, etc.

*Reference*: Totally Nonnegative Matrices, by S. Fallat and C. Johnson, Princeton University Press, 2011. (Note: Chapter 1 is freely available from the publisher's website)

# 3 Contributed talks

## Partial boundary value problems on finite networks

**Cristina Araúz**, Ángeles Carmona and Andrés M. Encinas
Universitat Politècnica de Catalunya

*Inverse boundary-value problems* were born to answer the question of whether it is possible to determine the conductivity of a body by means of boundary measurements. These problems are exponentially ill-posed since its solutions are highly sensitive to changes in the boundary data. We are mainly interested on the discrete version of the problem, that is, the *inverse boundary-value problems on finite weighted networks*. The aim here is to study *partial* inverse boundary-value problems, which are characterized by the existance of a part of the boundary where no data is known.

Given a weighted network with conductances on the edges $\Gamma = (V, c)$, we fix a proper and connected subset $F \subset V$ and will consider a certain kind of boundary value problems in which the values of the functions and of their normal derivatives are known at the same part of the boundary of $F$ and there exists another part of the boundary where no data is known. We determine when there is existance and/or uniqueness of solution on $\bar{F}$. For, it is mandatory to consider the Dirichlet-to-Neumann map of the network, its kernel and a local inverse of the matrix given by this kernel. We also observe that the kernel of the Dirichlet-to-Neumann map is a Schur Complement of the Schrödinger operator of the network.

## On permanents of Sylvester Hadamard matrices

**José Andrés Armario**
University of Seville

It is well-known that a Sylvester Hadamard matrix can be described by means of a cocycle. In this talk we show that the additional internal structure in a Sylvester Hadamard matrix, provided by the cocycle, is sufficient to guarantee some kind of reduction in the computational complexity of calculating its permanent.

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## Solving commutator equations in the groups of triangular matrices over a field

**Agnieszka Bier**
Silesian University of Technology, Poland

Let $K$ be a field. By $T_n(K)$ we denote the group of all invertible upper triangular matrices of size $n \times n$ over field $K$ and by $UT_n(K)$ we denote the subgroup of $T_n(K)$ consisting of all unitriangular matrices (i.e. matrices with all diagonal entries equal to 1).

A *basic commutator* $c_2(x_1,x_2) = [x_1,x_2]$ of elements $x_1$ and $x_2$ is defined as a product: $[x_1,x_2] := x_1^{-1}x_2^{-1}x_1x_2$. Let $d_2(x_1,x_2) = e_2(x_1,x_2) = c_2(x_1,x_2)$. Then for $i = 2,3,...$ we introduce:

$$\begin{aligned} c_{i+1}(x_1,\dots,x_{i+1}) &= [c_i(x_1,...,x_i),x_{i+1}], \\ d_{i+1}(x_1,\dots,x_{2^i}) &= [d_i(x_1,\dots,x_{2^{i-1}}),d_i(x_{2^{i-1}+1},...,x_{2^i})], \\ e_{i+1}(x_1,x_2) &= [x_1,\underbrace{x_2,x_2,...,x_2}_{i}]. \end{aligned}$$

In the talk we discuss solvability of certain commutator equations in $T_n(K)$ and $UT_n(K)$ with respect to the characteristic of field $K$. Namely, for a given triangular (respectively unitriangular) matrix $A$, we investigate the equations:

$$\begin{aligned} A &= c_i(X_1,X_2,...,X_i), \\ A &= d_i(X_1,X_2,...,X_{2^{i-1}}), \\ A &= e_i(X_1,X_2) \end{aligned}$$

in terms of existence of triangular (respectively unitriangular) matrices $X_1, X_2, ...$ for which the above equalities hold. We solve the problem within the wide range of cases.

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## A New Infinite Family of Two-Weight Codes

**Eimear Byrne**
University College Dublin

Projective two-weight codes over a finite field can be identified with sets of points in projective space and strongly regular graphs. Several constructions of such codes come from finite geometry. Two-weight codes over finite rings also give strongly regular graphs, but the geometric connection is more tenuous. In this talk we give a construction for a new infinite family of two-weight codes over a finite ring that determines a strongly regular graph whose complement has the same parameters as one defined from a hyperoval in PG(2,q).

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## The nonnegative inverse eigenvalue problem

**Anthony G. Cronin** and Thomas J. Laffey
University College Dublin

In this talk we investigate the *nonnegative inverse eigenvalue problem (NIEP).*
This is the problem of characterizing all possible spectra of nonnegative $n \times n$ matrices.
In particular, given a list of $n$ complex numbers $\sigma = (\lambda_1, \lambda_2, \dots, \lambda_n)$, can we find necessary and sufficient conditions on the list $\sigma$ so that it is the list of eigenvalues of an entrywise $n \times n$ nonnegative matrix $A$. The problem remains unsolved and a complete solution is known only for $n \leq 4$.
I will give a brief outline of the history and motivation behind the NIEP and discuss the more important results to date.
I will then give a survey of the main results from my PhD thesis and current work.

## The trace form and linear spaces of nilpotent matrices

**James Cruickshank** and Rachel Quinlan
National University of Ireland, Galway

We will consider linear spaces of n x n matrices, all of whose elements are nilpotent. In a celebrated series of papers Murray Gerstenhaber proved (among many other results) that the maximal dimension of such a space is $n(n-1)/2$. He also showed that any space of maximal dimension is conjugate to the space of strict upper triangular matrices. However, his proof imposed some restrictions on the order of the field. We will discuss a new proof of these old results which works over any field.

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## Miniversal deformations and codimensions of skew-symmetric matrix pencils

**Andrii Dmytryshyn**, Stefan Johansson, and Bo Kågström
Umeå University, Sweden

For each skew-symmetric matrix pencil $A - \lambda B$ we give:

- a miniversal deformation under congruence; that is, a normal form with minimal number of independent parameters to which all pencils $(A+E) - \lambda(B+F)$ close to $A - \lambda B$ can be reduced by transformations

$$(A+E) - \lambda(B+F) \mapsto \mathscr{S}^T(A+E)\mathscr{S} - \lambda \mathscr{S}^T(B+F)\mathscr{S},$$

  where $\mathscr{S} \equiv \mathscr{S}(E,F)$ smoothly depends on the entries of $E$ and $F$ and $\mathscr{S}(0,0) = I$.

- the general solution of the homogeneous system of matrix equations $(X^TA + AX, X^TB + BX) = (0,0)$, associated with the skew-symmetric matrix pencil $A - \lambda B$.

Both the miniversal deformations and the general solution allow us to calculate the codimension of the orbit of $A - \lambda B$ under congruence.

The motivation for these questions come from the open problem of the stratifications of skew-symmetric pencil orbits (i.e., constructing its closure hierarchies) under congruence transformations.

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## Automorphisms of the Sylvester matrix and related designs

**Ronan Egan**
National University of Ireland, Galway

We review the notions of regular and centrally regular group actions for pairwise combinatorial designs. Motivated by de Launey and Stafford's classification of the centrally regular subgroups for the Paley matrices, we discuss some (old and new) results in this area concerning the Sylvester Hadamard matrix and an equivalent design introduced by Kantor.

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## Solving large linear systems

**Waldemar Holubowski** and Dariusz Kurzyk
Institute of Mathematics
Silesian University of Technology, Gliwice, Poland

Large linear systems can be solved using known algorithms. However, if the coefficient matrix has a special form, it is possible to find much more efficient algorithms. In our talk we will consider the case where the coefficient matrix is triangular and banded. We explain how to solve such system with use of parallel computations.

## Linear algebra in communications

**Ted Hurley**
National University of Ireland, Galway

Why are Linear Algebra systems important in, in fact vital to, communications' systems? The talk will try to answer this question with various systems and examples.

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## Systems of linear and semilinear mappings

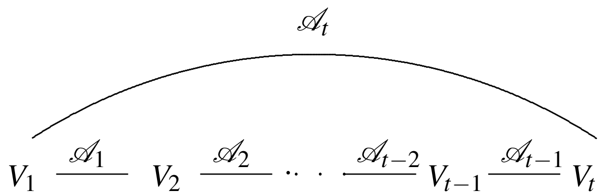
D. Duarte de Oliveira, R.A. Horn, **T. Klimchuk** and V.V. Sergeichuk
Kiev National Taras Shevchenko University

A mapping $\mathscr{A}$ from a complex vector space $U$ to a complex vector space $V$ is *semilinear* if

$$\mathscr{A}(u+u') = \mathscr{A}u + \mathscr{A}u', \qquad \mathscr{A}(\alpha u) = \bar{\alpha}\mathscr{A}u$$

for all $u, u' \in U$ and $\alpha \in \mathbb{C}$. We write $\mathscr{A} : U \to V$ if $\mathscr{A}$ is a linear mapping and $\mathscr{A} : U \dashrightarrow V$ (using a dashed arrow) if $\mathscr{A}$ is a semilinear mapping. A matrix of a semilinear operator $\mathscr{A} : U \dashrightarrow U$ is reduced by consimilarity transformations; recall that two matrices $A$ and $B$ are *consimilar* if there exists a nonsingular matrix $S$ such that $\bar{S}^{-1}AS = B$.

In [3] we give a canonical form of matrices of a *cycle* of linear and semilinear mappings




in which each line is a full arrow $\longrightarrow$ or $\longleftarrow$, or a dashed arrow $\dashrightarrow$ or $\dashleftarrow$. We extend to such systems Paul Van Dooren's regularizing algorithm for the computation of all irregular summands in Kronecker's canonical form of a matrix pencil, which uses only unitary transformations (his algorithm was extended in [4] to cycles of linear mappings).

Gelfand and Ponomarev proved that the problem of classifying pairs of commuting linear operators contains the problem of classifying $k$-tuples of linear operators for any $k$. We extended in this statement to semilinear mappings: The problem of classifying pairs of commuting semilinear operators contains the problem of classifying $(p+q)$-tuples consisting of $p$ linear operators and $q$ semilinear operators, in which $p$ and $q$ are arbitrary nonnegative integers.

## References


[1] I.M. Gelfand, V.A. Ponomarev, Remarks on the classification of a pair of commuting linear transformations in a finite dimensional vector space, Funct. Anal. Appl. 3 (1969) 325–326.

[2] D. Duarte de Oliveira, R.A. Horn, T. Klimchuk, V.V. Sergeichuk, Remarks on the classification of a pair of commuting semilinear operators, Linear Algebra Appl. 436 (2012) 3362–3372.

[3] D. Duarte de Oliveira, V. Futorny, T. Klimchuk, D. Kovalenko, V.V. Sergeichuk, Cycles of linear and semilinear mappings, 2012, arXiv:1208.5418.

[4] V.V. Sergeichuk, Computation of canonical matrices for chains and cycles of linear mappings, Linear Algebra Appl. 376 (2004) 235–263.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### Multidimensional matrices uniquely recovered by their lines


**Joon Yop Lee**
POSTECH, Korea


We provide a method to determine if a $q$-ary multidimensional matrix is lonesum or not by using properties of line sums of lonesum multidimensional matrices. In particular, we establish a graphic method that uses edge-colored graphs to determine if a binary multidimensional matrix is lonesum or not. We also provide two methods to determine if a $q$-ary multidimensional matrix is lonestructure or not. The first one uses properties of line structures of lonestructure multidimensional matrices and the second one uses edge-colored directed multigraphs.

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### Norms of 0–1 Schur multipliers


**Rupert H. Levene**
University College Dublin


A Schur multiplier is a linear map

$$S_B\colon \mathbb{C}^{m\times n} \to \mathbb{C}^{m\times n}, \quad A \mapsto A \bullet B$$

where $B \in \mathbb{C}^{m\times n}$ is a fixed matrix and $\bullet$ is the Schur product, which is also known as the Hadamard product. Let $\|A\|$ be the largest singular value of $A \in \mathbb{C}^{m\times n}$ and consider the norm

$$\|S_B\| := \sup_{\|A\|\leq 1} \|A \bullet B\|.$$

Livschits [1] has proven that if the entries of $B$ are all 0s and 1s, then $\|S_B\|$ cannot lie in either of the "gaps" $(0,1)$ or $(1,\sqrt{4/3})$. By calculating the norms of some specific Schur multipliers and using simple combinatorial arguments, we show that that there are at least four more gaps in the set of norms of 0–1 Schur multipliers.

## References


[1] Leo Livschits, *A note on 0-1 Schur multipliers*, Lin. Alg. Appl. **22** (1995), 15–22.

### Completions of Partial Matrices

**James McTigue** and Rachel Quinlan
National University of Ireland, Galway

A partial matrix over a field $\mathbb{F}$ is a matrix whose entries are either elements of $\mathbb{F}$ or independent indeterminates. A completion of such a partial matrix is obtained by specifying values from $\mathbb{F}$ for the indeterminates. We determine the maximum possible number of indeterminates in an $m \times n$ partial matrix ($m \leq n$) whose completions all have a particular rank $r$, and we fully describe those examples in which this maximum is attained, without any restriction on the field $\mathbb{F}$.

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### A new technique to extract invariants of matrix groups

**Alexander D. Rahm**
NUI Galway
funded by the Irish Research Council

We introduce a method to explicitly determine the Farrell-Tate cohomology of discrete groups. This method has very recently been applied to the Coxeter triangle and tetrahedral groups as well as to the Bianchi groups, i.e. $PSL_2$ over the ring of integers in an imaginary quadratic number field. Our method has allowed to show that the Farrell-Tate cohomology of the Bianchi groups is completely determined by the numbers of conjugacy classes of finite subgroups. In fact, our access to Farrell-Tate cohomology allows us to detach the information about it from geometric models for the Bianchi groups and to express it only with the group structure. Our new insights about the homological torsion allow us to give a conceptual description of the cohomology ring structure of the Bianchi groups.

### Combinatorial bases for the generalized null space and height and level characteristic of $M_\vee$-matrices

**Manideepa Saha** and Sriparna B.
Indian Institute of Technology Guwahati
Guwahati, Assam, India

An $M_\vee$- matrix is one which can be written as $A = sI - B$ with $s > 0$, $\rho(B) \leq s$ and $B$ an eventually nonnegative matrix, that is for some positive integer $l$, $B^k \geq 0$ for ,all $k \geq l$. In this paper we studied the combinatorial structure and graph theoretical structure of the generalized null space of $M_\vee$-matrices. We obtained results on height and level characteristics, and established some equivalent results for the equality of these characteristics.

## References

[1] D.D.Olesky, M.J.Tsastsomeros, P.Van Den Driessche, $M_\vee$-matrices: A generalization of $M$-matrices based on eventually nonnegative matrices, *ELA* 18:339–351,2009

[2] D. Hershkowitz, and H. Schneider, *Height bases, level bases, and the equality of the height and the level characteristics of an M-matrix*, Lin. Multilin. Algebra,25, 149–171, 1989.

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### Triangularizing Matrix Polynomials

**Leo Taslaman**
School of Mathematics, The University of Manchester

This talk concerns the following problem: given a matrix polynomial $P(\lambda)$, can we build a triangular matrix polynomial having the same finite and infinite elementary divisors, the same size and the same degree as $P(\lambda)$? We show that over the complex numbers the answer is always yes. Over the real numbers it is not always possible to construct such triangular matrix polynomial. However, we show that we can always build a quasi-triangular matrix polynomial. Finally, we rephrase what we have done in terms of inverse polynomial eigenvalue problems and conjecture the following statement: every regular real matrix polynomial is equivalent to a regular Hermitian matrix polynomial and vice versa.

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## Critical Exponents: Old and New

**Olivia Walch**
University of Michigan, USA

Let $\mathscr{P}$ be a class of matrices, and let $A$ be an $m$-by-$n$ matrix in the class; consider some continuous powering, $A^{\{t\}}$. The critical exponent of $\mathscr{P}$, if it exists, with respect to the powering is the lowest power $g(\mathscr{P})$ such that for any matrix $B \in \mathscr{P}$, $B^{\{t\}} \in \mathscr{P}\ \forall\ t > g(\mathscr{P})$. For powering relative to matrix multiplication in the traditional sense, hereafter referred to as *conventional* multiplication, this means that $A^t$ is in the specified class for all $t > g_C(\mathscr{P})$. For Hadamard multiplication, similarly, $A^{(t)}$ is in the class $\forall\ t > g_H(\mathscr{P})$. We consider two questions for several classes $\mathscr{P}$ (including doubly nonnegative and totally positive): 1) does a critical exponent $g(\mathscr{P})$ exist? and 2) if so, what is it? For those where no exact result has been determined, lower and upper bounds are provided.

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# 4 Irish Meeting for Linear Algebra Research : Saturday December 8

Room AC201, Main Concourse, NUI Galway

### Finite rings with many commuting pairs of elements

**Stephen Buckley**, Des MacHale and Áine Ní Shé
National University of Ireland, Maynooth

We investigate the set of values attained by the probability $\Pr(R)$ that a random pair of elements in a finite ring $R$ commute. Specifically, we characterize all values greater than or equal to 11/32. We also define a notion of isoclinism for rings, and show that each $\Pr(R)$ value strictly greater than 11/32 uniquely identifies the isoclinism class of $R$, whereas $\Pr(R) = 11/32$ arises from five distinct isoclinism classes. Most of the proof involves linear algebra over $\mathbb{Z}_p$, where p is prime.

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### Modelling Cumulative Mortality Data and a Surprising Consequence of Matrix Algebra

**John Hinde**
Statistics, School of Mathematics, Statistics and Applied Mathematics,
NUI Galway

In toxicological experiments for biological pest control, experiments frequently involve the study of cumulative mortality in a groups of insects measured at various time points. Samples of different strains, or isolates, of the compound under study are used, typically with some replication. The example considered here is from a study of a microbial control to insect damage in sugar cane. A special form of regression model based on a cumulative multinomial distribution provides an approach to the analysis of these data, however, the basic model needs to be extended to account for extra-variation. This can be easily done by including additional random effects in the regression model and a modified form of least-squares can be used to obtain parameter estimates. Surprisingly a number of apparently different models give identical results. Why should this be so? A careful study of the form of the models and a result from matrix algebra provide the clue.

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### Matrix theory, number theory, and quantum spin chains

**Stephen Kirkland**
National University of Ireland, Maynooth

The time–evolution of quantum spin systems models the dynamics of various nanodevices, including “quantum buses” for transferring information in quantum processors. One of the desirable tasks is to transfer the state of a particle into another one with maximum fidelity; when that fidelity is 1, we have *perfect state transfer*. In mathematical terms, this is equivalent to considering an undirected graph with adjacency matrix $A$, and asking for a particular entry of the matrix $e^{itA}$ to have modulus 1 at some time $t$.

It is known that for spin chains (where the graph in question is a path), perfect state transfer occurs between the end points of the chain only if the number of vertices is small. In view of that fact, we consider the following relaxation of perfect state transfer: if, for each $\varepsilon > 0$, there is a $t > 0$ such that the fidelity at time $t$ exceeds $1 - \varepsilon$, then we say that *pretty good state transfer* occurs. Using techniques from matrix theory and number theory, we show that a spin chain on $n$ vertices exhibits pretty good state transfer between the end points of the chain if and only if either $n+1$ is a power of 2, or for some prime $p$, we have $n+1 = p$ or $n+1 = 2p$.

## The symmetric nonnegative inverse eigenvalue problem

**Thomas Laffey**
University College Dublin

Let $\sigma := (\lambda_1, \lambda_2, \ldots, \lambda_n)$ be a list of real numbers. The symmetric nonnegative inverse eigenvalue problem (SNIEP) asks for necessary and sufficient conditions for the existence of a symmetric matrix with nonnegative entries and spectrum $\sigma$. If this occurs, $\sigma$ is said to be symmetrically realizable.

In terms of $n$, progress on the problem has been slow. For $n \leq 4$, a complete solution was found by Loewy and London in 1978.

The case when $n = 5$ and $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 = 0$ was solved by Spector in 2010.

A major obstruction to progress has been the lack of constructive methods, as given a monic polynomial with real roots, there is no simple algorithm for constructing a symmetric matrix with that characteristic polynomial, analogous to the companion matrix in the general case.

We will present a survey of the known results on SNIEP, with a particular emphasis on new constructive techniques. In particular, a method, first presented by Šmigoc in her PhD thesis, has been developed and refined in joint work with her to provide a uniform approach to most of the known results. The question of the effect on symmetric realizability of adding zeros to the list $\sigma$ will also be discussed.

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## Companion type matrices in the nonnegative inverse eigenvalue problem

Thomas Laffey and **Helena Šmigoc**
University College Dublin

The question, which lists of complex numbers are the spectrum of some nonnegative matrix, is known as the nonnegative inverse eigenvalue problem (NIEP). In this talk we will give an overview of some of the recent constructive approaches the NIEP with the emphasis on the constructions that use the companion type matrices.

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